(4.4.s1) Remark. We assumed above that the generator $G$ was bounded, i.e. $\sup_{i,j \in S} |g_{ij}| < \infty$. If this condition is violated, then strange behaviour can occur$^1$. For example, suppose $S = \{1, 2, 3, \ldots\}$, with $g_{i,i+1} = 2^i$ and $g_{ii} = -2^i$ for all $i \in S$, and $g_{ij} = 0$ otherwise. Then this process can only increase, by 1 each time. Furthermore, the expected time to increase from $i$ to $i+1$ is equal to $1/g_{i,i+1} = 2^{-i}$. Hence, starting from $X_0 = 1$, the expected time to increase all the way to infinity is equal to $\sum_{i=1}^{\infty} 2^{-i} = 1$. That is, in a finite time (equal to one second, on average), the process will escape all the way to infinity, and hence “vanish”. This property is called being explosive. Clearly, for such processes, formulas for $P^{(t)}$ such as (4.4.7) no longer hold.

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