The general Wishart distribution is available from the equation above. The equation gives a parameter transform of the distribution in the standardized case. The standardized case has its distribution available by a simple geometrical-regression analysis argument: $(x_{11}, \ldots x_{1,n-1})$ has length with a χ_{n-1} distribution. $(x_{21}, \ldots x_{2,n-1})$ has a component in the direction $(x_{11}, \ldots x_{1,n-1})$ and this component has a standardized normal distribution; it has a component perpendicular to this with length having a χ_{n-2} distribution. And so on for trivariate and multivariate case — components in the orthogonal frame built up by earlier components have standardized normal distributions — the remainder has a χ distribution.

On November 28th I gave a fiducial talk at Berkeley and it was received with utter contempt by Neyman and bare tolerance by the others. . . .

D.A.S. Fraser to Fisher: 10 *January* 1962

I hope my letter of Jan 6 reached you in India. I have now heard from Keith Hastings at Toronto and shall quote several of his formulae — they tie in with those in my Jan 6 letter. [With] $g_{ii} = \chi_{n-i}$, $g_{ij} = N(0,1)$, $[i \neq j$, and all $g_{ij}]$ independent,

$$\begin{pmatrix} s_1 & s_2 r_{12} & s_3 r_{13} \\ 0 & s_{2.1} & s_{3.1} r_{23.1} \\ 0 & 0 & s_{3.12} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ 0 & g_{22} & g_{23} \\ 0 & 0 & g_{33} \end{pmatrix} \begin{pmatrix} \sigma_1 & \sigma_2\rho_{12} & \sigma_3\rho_{13} \\ 0 & \sigma_{2.1} & \sigma_{3.1}\rho_{23.1} \\ 0 & 0 & \sigma_{3.12} \end{pmatrix}$$

[i.e.] $\mathbf{S} = \mathbf{G}\Sigma$ say. Let $\mathbf{D(S)}$ for example be:

$$\begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_{2.1} & 0 \\ 0 & 0 & s_{3.12} \end{pmatrix}$$

The pivotal equation is $\Sigma\mathbf{D}^{-1}(\Sigma) = \mathbf{G}^{-1}\mathbf{S}\mathbf{D}^{-1}(\mathbf{S})\ \mathbf{D}(\mathbf{G})$ which yields

$$\frac{s_2 r_{12}}{s_{2.1}} g_{22} = \frac{\sigma_2\rho_{12}}{\sigma_{2.1}} g_{11} + g_{12},$$

$$\frac{s_3 r_{13}}{s_{3.12}} g_{33} = \frac{\sigma_3\rho_{13}}{\sigma_{3.12}} g_{11} + \frac{\sigma_{3.1}\rho_{23.1}}{\sigma_{3.12}} g_{12} + g_{13},$$

$$\frac{s_{3.1} r_{23.1}}{s_{3.12}} g_{33} = \frac{\sigma_{3.1}\rho_{23.1}}{\sigma_{3.12}} g_{22} + g_{23}.$$

Hastings then quotes the following probability density function for r_{12}, r_{13}, r_{23}:

$$K\,D_r^{-(n+3)/2}\,D_\rho^{-(n-1)}\{(1-\rho_{12}^2)(1-\rho_{13}^2)(1-\rho_{23}^2)\}^{(n-1)/2} \times \int\int_0^\infty\int (uvw)^{n-2} \times$$

$$\exp\{-\tfrac{1}{2}(u^2+v^2+w^2-2uvr_{12}\rho_{12.3}-2vwr_{23}\rho_{23.1}-2uwr_{13}\rho_{13.2})\}\,du\,dv\,dw,$$

$$\text{where } D_r = \begin{vmatrix} 1 & r_{12} & r_{13} \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{vmatrix}$$

and this has the form in your Nov 29 letter with a difference in power for (uvw).

Solving the pivotal equation the other way for the ρ's yields acc. to his letter: p.d.f. for $\rho_{12}\,\rho_{13}\,\rho_{23}$

$$K\,D_r^{-(n-3)/2}\,(1-r_{12}^2)^{-1}\,\{(1-\rho_{12}^2)(1-\rho_{13}^2)(1-\rho_{23}^2)\}^{(n-2)/2} \times$$

$$D_\rho^{-(n+1)}\,(1-\rho_{12}^2)^{-1}(1-\rho_{13.2}^2)^{-1}\int\int_0^\infty\int u^n v^{n-2} w^{n-4}\exp\{\text{as above}\}\,du\,dv\,dw.$$

This seems asymmetric generally; definitely is for $r_{12} = r_{13} = r_{23}$. This distribution relates the ρ's to the r's by reference to a definite order in which regression is run on the variables 1,2,3. . . .

Fisher to D.A.S Fraser: 11 *January* 1962

I have just seen your letter of Jan 6. Do not forget to look up Walter Bodmer, who has also had some experience of being 'bawled down' by Neymanians. They intimidate Americans successfully enough, especially refugees anxious to get posts in American Universities. I do not think they need intimidate anyone else. For your encouragement I transcribe the first paragraph of a letter just received from a mathematical logician[1] at the Rockefeller Institute.

'I have just finished your book *Statistical Methods and Scientific Inference.* I wish I had seen it sooner. I am just delighted with it, for I have felt altogether alone in my dire suspicions of the logical inadequacy of the theory of statistical inference as expounded by Neyman, Pearson and nearly every other statistician I can think of . . .'. Later on, he says 'I am interested in tracking down everything that could possibly be of use to me concerning fiducial inference; for this seems to me to be the fundamental form of statistical inference — the decision theoretic approach being etc.'

I think I told you I had the simultaneous distribution of r_{ij} given ρ_{ij} for t variates. It can be written (if I have room)

$$\frac{\pi^{-t(t-1)/4}\,2^{-t(N-3)/2}}{\{(N-3)/2\}!\ldots\{(N-t-2)/2\}!}\,|\rho_{ij}^*|^{(N-1)/2}|r_{ij}|^{(N-t-2)/2}\,dr_{ij}F_{N-2}(\gamma_{ij})$$