CAPITALISM'S GROWTH IMPERATIVE

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ABSTRACT

A capitalist firm operating in a competitive market is subject to a growth imperative, because uncertainty about the profit rate under a no-growth policy makes the firm's prospects highly unattractive in finite time and bankruptcy practically certain in the long run. A no-growth policy determines consumption and investment so that they and capital would remain constant over time if the latter's expected return were realized with certainty. Simulation is used to arrive at the probability of bankruptcy by the end of t periods and the expected values of capital and money, for relevant combinations of time and uncertainty under successively more realistic models of a no-growth firm in a competitive market. The sensitivity of the results to variation in the parameters in each of the models is evaluated. Finally, we consider what firms might actually do to achieve the benefits of growth and more attractive prospects of survival.

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ABSTRACT

A capitalist firm operating in a competitive market is subject to a growth imperative, because uncertainty about the profit rate under a no-growth policy makes the firm’s prospects highly unattractive in finite time and bankruptcy practically certain in the long run. A no-growth policy determines consumption and investment so that they and capital would remain constant over time if the latter's expected return were realized with certainty. Simulation is used to arrive at the probability of bankruptcy by the end of t periods and the expected values of capital and money, for relevant combinations of time and uncertainty under successively more realistic models of a no-growth firm in a competitive market. The sensitivity of the results to variation in the parameters in each of the models is evaluated. Finally, we consider what firms might actually do to achieve the benefits of growth and more attractive prospects of survival.

INTRODUCTION

The primary purpose of this paper is to establish that a capitalist enterprise operating in a competitive environment, be it a proprietorship or a corporation, is subject to a growth imperative. By a growth imperative, we mean that the enterprise requires the expectation of a positive growth rate, probably one that is well above the physically feasible rate of growth for an actual closed competitive capitalist system. A positive and probably high mean rate of growth is necessitated by the facts that the actual profit rate is uncertain, and its realization varies over a very wide range. This high variability makes an enterprise with a zero or negative mean expected value for its growth rate face a future in which bankruptcy is practically certain in the long run, and has an intolerably high probability in the short run, while providing little or no compensating benefits by way of growth in income or wealth until bankruptcy takes place. In fact, income and wealth can be expected to fall over time. By a competitive capitalist enterprise, we mean one in which the enterprise buys and sells in markets in which government or other regulatory authority plays no role in determining the prices and quantities for what it buys and sells. The reliance on competitive markets for specialization and exchange and for the determination of each capitalist's profit are the source of the high variability in the profit rate and the necessity of a positive and probably high mean expected rate of growth.

It is widely if not universally accepted that growth is desirable. Some environmentalists fear that growth per se is destroying our physical environment. Representatives of third world countries have charged that policies which generate growth in the industrial countries are at the expense of their people, and some development economists express the fear that this may be true. However, with few exceptions, economists in developed capitalist countries take for granted that growth is desirable and their concern is with policies that achieve growth. Keynes gave rise to a macroeconomic theory under which investment is not only desirable for growth in the long run, but necessary to avoid stagnation and unemployment in the short run.

We make the stronger claim here that growth is not merely desirable: it is necessary for tolerable prospects for future survival for each capitalist and perhaps for the system as a whole. To our knowledge, only some business leaders make the same claim on the micro level. But they argue that innovations in technology, marketing, etc. by their competitors force them to participate in the quest for competitive advantage. To our knowledge, no theory of a capitalist firm or individual has been advanced under which a policy intended to maintain it at a stationary level is certain or even highly likely to lead to its collapse in the long run and offer highly unattractive prospects in the short run.

Neoclassical theory, mainstream economic theory, offers a radically different view of a capitalist firm. Nothing is easier for a firm than remaining in a stationary state, and for a real person, growth is a matter of taste. It is no more than preference between present and future consumption.

We say it is capitalism that is subject to a growth imperative, because the same is not true of a pure socialist system. Regardless of what other problems are faced by a socialist system, its own operation does not subject it to a growth imperative. It will be seen that notwithstanding a common technology of production, the difference between how it is administered in the two systems give rise to a growth imperative in one and its absence in the other. The propositions that a pure socialist system offers security and stagnation, while a pure (competitive) capitalist system offers insecurity and growth have been recognized, but they have not been investigated rigorously. What we will do here is to explain rigorously how a capitalist system generates insecurity and growth, and then consider what capitalists do to deal with the insecurity.

The next section presents a simple model of a capitalist firm and establishes what happens to its probability of survival and its wealth over time under a no-growth policy, both conditional on its survival and without that condition. We find that it is certain to go bankrupt in the long run, and simulation of the model reveals that the firm does poorly in finite time *regardless* of the profitability and other relevant variables. Section II reviews the treatment of growth, uncertainty and bankruptcy in the economics literature. Prior to Keynes, the problem of uncertainty about the future was resolved by assuming that the uncertain future value of a variable could be represented by its mean. Keynes's dissatisfaction with this solution to the investment problem motivated a postwar literature on investment under uncertainty and risk aversion. The mainstream theoretical literature established that the utopian properties of a perfectly competitive capitalist system are preserved in the face of uncertainty and risk aversion. This was accomplished and the feasibility of a stationary state was maintained, we shall see, by ignoring or trivializing the problem of bankruptcy.

Section III introduces a number of refinements in the model that make it more realistic.

Bankruptcy in the long run is not shown to be certain in every case, but its probability is very high for combinations of uncertainty and time that are large. Finally, Section IV establishes that investment and expenditure policies to achieve growth realize that objective, but they do not increase the probability of survival. Far more is needed.

One of the main reasons for interest in microeconomic models of a firm is their use as a foundation for macro models of a capitalist system. The hope is that the macro model will illuminate the short-term fluctuations and long-term development of actual capitalist systems. We have that hope for the models of a capitalist firm presented here. More needs to be done, however, before moving to the macro level with these models. What we can and will do, in the section on growth and in the conclusion, is raise questions about the performance of actual capitalist systems that are suggested by our model.

I. A SIMPLE MODEL OF CAPITAL

This section will present a model of a capitalist, i.e., a proprietor or a corporation, that is simple, powerful, and, we believe, useful for understanding their behavior on the micro level and the system's macro behavior. The model incorporates real and nominal capital, uncertainty about the profit rate on capital, consumption, investment in both forms of capital, and bankruptcy, all in a plausible manner. There are also important omissions, as we shall see, among them labor, government and monetary policy.

THE MODEL:

The wealth of a capitalist, also called a firm in what follows, at the start of period t is comprised of its capital valued at cost, K(t), and the nominal amount of its net monetary assets or money, M(t). Their sum is his wealth,

$$W(t) = K(t) + M(t). \quad (1)$$

The initial value of K(t), K(1), is given, and the change in K(t), from one period to the next is given by

$$K(t+1) = K(t)[1 - \lambda] + I(t), \quad (2)$$

where λ = rate at which capital depreciates in productivity from one period to the next,

and I(t) = expenditure of money on additions to the gross stock of capital.

The firm's quantity of net monetary assets is its cash, receivables and bonds, less its payables and debt. M(1) is given and M(t+1) is raised above M(t) by M(t)r, the interest on its initial balance and by P(t), the gross profit on production. M(t+1) falls as a consequence of the period's investment and C(t), the consumption or dividend plus overhead costs incurred by the firm. Hence,

$$M(t+1) = M(t)[1 + r] + P(t) - C(t) - I(t). \tag{3}$$

The actual gross profit on production during t is

$$P(t) = \alpha(t)K(t), \tag{4}$$

with $\alpha(t)$ assumed to be a normally distributed random variable with mean μ and standard deviation σ.

The capitalist's consumption plus administration cost is given by the expression:

$$C(t) = \eta K(t),$$

(5)

and the gross investment in capital is

$$I(t) = g\lambda K(t).$$

(6)

Here, g is the ratio of the investment expenditure to the depreciation on the capital, so that g=0 means that gross investment is zero and g=2 means that investment in t is twice the depreciation on the existing stock.

The economic security of a firm increases with the ratio M(t) / K(t), and insecurity rises as M(t) / K(t) falls below zero. There is a lower limit to the ratio; that limit is no lower than M/K = -1, and when that limit is reached, the firm is bankrupt.[1] The creditors then take and liquidate the firm's assets for whatever can be recovered of the money owed to them. The corporation ceases to exist, and the proprietor ceases to be a capitalist. The upper limit on -M/K is in fact less than one, due to the loss on and costs of taking over and liquidating the assets of a bankrupt firm. This condition for bankruptcy may seem to be excessively harsh, and today bankruptcy law is far kinder to firms in financial distress, as will be seen later. However, this seemingly harsh policy would be the practice in a competitive capitalist system, where the law does not

1 A widely used measure of financial position is the debt-equity ratio, -M(t) / [K(t) + M(t)]. We could just as well have used it as our bankruptcy condition.

overrule private contracts.[2]

THE NO-GROWTH POLICY

To establish that a firm operating in competitive markets is subject to a growth imperative, let us first recall what economists mean by "competitive markets." The firm buys the labor and materials employed in production in markets where there are many buyers and sellers, so that each firm has practically no influence on the prices paid for its factors of production. The same is true of the prices at which its products are sold. The conditions of supply and demand fluctuate over a wide range in the markets where the firm buys and sells, so that the gross margin per unit of product and the volume of sales are both subject to wide non-compensating variation from one period to the next. The consequence is that the gross profit on production, that is, earnings before deducting administration or overhead expenses, depreciation and interest, expressed as a fraction of capital employed by the firm can fluctuate over a wide range from one period to the next. This is captured by the mean and variance of $\alpha(t)$ in Eq. (4).

The basis for our claim that a capitalist enterprise operating in competitive markets is subject to a growth imperative is that a no-growth policy makes bankruptcy practically certain in the long run and have an intolerably high probability in the short run, while providing little or no compensating benefits. We define a no-growth policy as one under which C and I are determined so that the values of K and M would remain constant over time if the expected value

2 Understanding the model may be helped by assuming that you have purchased a house at the start of $t = 1$ for $K(t) = \$100,000$ with the help of a mortgage in the amount of $M(1) = -\$50,000$. If you rent the house to others, the rental income in year t is $\alpha(t)K(t)$, and if you live in the house $\alpha(t)K(t)$ is your employment income. The mortgage is reduced (M is increased) by $\alpha(t)K(t)$ in year t, and the mortgage is increased by your consumption $\eta K(t)$, by your expenditure to maintain the house $g\lambda K(t)$, and by the interest on the mortgage. The terms of the mortgage also stipulate that if and when it rises to ninety percent of K(t), the bank takes possession of the house, and the bank, not you, realizes the gain or loss on the difference between the sale price of the house and M(t). The model does not capture all of the rich detail of the actual world, but it goes a long way in that direction.

of the profit rate were realized with certainty. That policy is realized under the model represented by the above equations when the expenditure rate

$$q = g\lambda + \eta = \mu, \text{ and } g = 1 \text{ and } r = 0. \tag{7}$$

The condition that g = 1 guarantees that K remains constant over time by making investment equal to depreciation. The condition that $q = \mu$ makes the expenditure in each period equal to the average profit rate. Hence, with r = 0, we have C, I and M as well as K constant over time when the future is certain, that is, when $\alpha(t) = \mu$ for all t.

With the Eqs. (7) true, the model satisfies a martingale property, so that the firm is *certain* to go bankrupt in the long run. This is demonstrated in the Appendix (Theorems 3 and 4).[3] An explanation of why bankruptcy is inevitable sooner or later that may be easier to understand and somewhat oversimplified is possible. Let bankruptcy take place when M/K = -1 or less and ignore for the moment the investment and consumption-administration expenditures. Note that the operating profit or loss for a period can be a loss of any magnitude, such as 300% of K or more. If M/K at the period's start is low enough so that a loss of 300% or more makes M/K = -1 or less, the firm is bankrupt. The likelihood of bankruptcy increases with the variance of the profit rate and inversely with M/K. Also, the likelihood it will be sooner rather than later is increased by the size of the periodic investment and consumption-administration expenditures as a fraction of K.

The proof that a capitalist enterprise is certain to go bankrupt in the long run may remind some readers of Keynes's response to the proof that the long-run tendency of a perfectly competitive capitalist system is full employment: "In the long run we are all dead." An actual

3 The appendix establishes that bankruptcy is certain in the long run under weaker assumptions than those stated above. In the course of what follows, we will develop more realistic models of the firm which will recognize r > 0 among other things, and we will consider their consequences for the no-growth policy and the firm's long run survival.

capitalist is more concerned with the probability of bankruptcy over a reasonable, finite time horizon.

SIMULATION OF MODEL:

To arrive at some idea of the consequences of a no-growth policy in finite time for a firm described by the above model, we simulated its fortunes. We arrived at the probability of bankruptcy by the end of t periods, and the expected values of K and M, both conditional on survival for t periods and without that condition. For each simulation, we calculated the probability of bankruptcy by the end of t periods for t = 2, 10, 25, 50 and 100. Bankruptcy was assumed to take place when the debt reaches -M(t) = .5K(t).[4] The calculations were made for σ = 0.0, .10, .20, .40, and .60. The simulation was carried out for a system with 10,000 firms, each one initially identical and with subsequent fortunes that differed only with respect to the random outcome with respect to α(t).

Table I presents the results of a simulation under which K(1) has the arbitrary value of \$1,000 and M(1) = 0. Regardless of the initial K(1) value, all that matters is M(1) relative to K(1). Reasonable values assigned to the relevant variables that satisfy the no-growth condition were depreciation rate λ = .10, consumption-administration rate η = .15, mean gross profit rate μ = .25, and investment ratio g = 1. The expenditure rate is q = gλ + η and with r = 0, the no-growth condition is satisfied.

We see in Table I that with σ = 0 and the future certain K = 1,000 and M = 0 in every

4 It may be wondered why bankruptcy takes place when -M(t) reaches .5K(t) instead of K(t). When -M(t) reaches .5K(t), the debt covenant allows the creditors to call their loans. The market value of the firm's capital will then be below K, its book value, so that the firm is unable to meet their demand or refinance the debt elsewhere, and it is wiped out. Making bankruptcy take place when -M(t) is closer to K(t) would not result in any substantive change in the conclusions reached. The critical assumption is that government does not intervene in credit markets to govern or overrule the terms of debt contracts in the interest of debtors.

future period. Also, gross investment $\lambda K = 100$ and consumption $\eta K = 150$ in every period. With $\sigma > 0$, the investment policy keeps the mean K = $1,000 as long as the firm survives, but that is only true conditional upon the firm surviving for the t periods. Most important, the probability of bankruptcy rises as σ rises above zero and as we go further into the future. For example, with $\sigma = .4$, the probability of bankruptcy rises above one-half by $t = 10$, and with $\sigma = .60$, the probability of bankruptcy rises to .84 by $t = 50$.

With the mean values of K and M over time conditional on survival, M rises sharply with σ as well as t, because the surviving firms are profitable and the no-growth policy limits them to putting the excess profits in the bank. The unconditional expected value of M also rises with both σ and time, but the unconditional values of wealth, that is $W = K + M$ fall for each value of $\sigma > 0$ as t rises. The important attributes of W are that it falls as t rises for each value of σ and it never rises above its initial value of $W(1) = 1{,}000$. Note how sharply the unconditional expected value of K falls with time.

We simulated the model represented by Eqs. (1) to (6) for a number of other combinations of the parameters that satisfy the no-growth conditions of Eq. (7). We were surprised to find that all of these produced exactly the same output that we see in Table I.[5] For instance, $\lambda = \eta = \mu = 0$ also satisfy the no-growth condition, and that simulation also results in exactly the same survival probabilities and K and M values as in Table I. We then found that to be no coincidence: Theorem I in the Appendix proves that it must take place.

UNCERTAINTY OF THE PROFIT RATE:

5This takes place when the random number generator that determines the sequence of profit rates for each firm starts at the same place as we go from one combination of the parameters to another. What happens when the random number generator starts at a randomly selected new place every time we run a simulation will be discussed later.

Table I makes clear that the probability of bankruptcy in finite time depends materially on σ, the uncertainty of the profit rate on capital. It would therefore be most desirable to have some idea as to the plausible range of σ. We have practically no direct data on that statistic, but a related statistic on which considerable data exists is the standard deviation of the return on the U.S. stock market in the past. The capitalization weighted return on all stocks traded on the NYSE over the years 1926 to 1990, had an arithmetic mean of 11.9 per cent, a geometric mean of 8.6 per cent, and a standard deviation of 21.1 per cent. See Siegel [1992, p.31]. Individual stocks had mean returns that were larger or smaller than 11.9 per cent, and their standard deviations were on average larger, since the return on the stock market as a whole benefited from diversification. Also, firms traded on the NYSE did not operate in competitive markets as described earlier. They all enjoyed varying degrees of monopoly power, acquired in order to raise gross return and reduce risk. Consequently, we may speculate that σ = .20 is very low for an average firm that buys and sells in competitive markets. They may well have returns with a standard deviation as high as or higher than .60, and .40 is a quite conservative figure.

A further consideration in reaching a judgement on our value of σ is that the upper limit on the loss on a share of stock over some time period is 100 per cent,[6] but there is no upper limit on a capitalist loss expressed as a fraction of the capital employed. To see why, recall that the rate

6This property of a stock has generated much interest in the comparative features of the arithmetic and geometric mean rates of return on a security. The arithmetic mean is an average of N simultaneous returns, while the geometric mean is the average rate at which wealth will grow over N periods. It is the annual return that one can expect to earn in the long run -- as time goes to infinity. The geometric mean is below the arithmetic mean to a degree that depends on the variance of the return, and if there is a non-zero probability of a return of -100%, a portfolio that consists solely of this security will go to zero in the long run. This property of the geometric mean return has generated interest in Kelly strategies, that is, portfolios that maximize the geometric mean and combining them with a risk-free asset to limit the probability of bankruptcy. See Markowitz [1976], and MacLean and Ziemba [1999]. There will be more on bankruptcy in the next section.

of profit (or loss) on a share of stock is the dividend plus the rise (fall) in price divided by the *initial price*. Since the dividend and price cannot fall below zero, the largest possible loss is 100 per cent. The rate of profit (loss) for an enterprise is the operating profit (loss) divided by the capital employed valued at its cost. Subject to the ability of the enterprise to draw on a cash reserve or borrow from a bank, the loss in any period can be a few hundred percent. One of the advantages attributed to private enterprise by comparison with government enterprise is that there is a limit on the ability of the former "to throw good money after bad money" or get others to join in doing so.

CONSEQUENCES:

The consequences of a no-growth policy for a capitalist described by Eqs. 1 to 6 are quite unattractive. As stated earlier, bankruptcy is certain in the long run, the unconditional expected values of total wealth decline over time. In finite time, the consumption and the stock of capital, conditional on survival, remain constant: only the stock of money grows, but its sole purpose is to defer bankruptcy, and it is not very effective for that purpose. The absence of growth in consumption and capital are an expected consequence of the no-growth policy, but there is no compensatory benefit in a high probability of survival in finite time as well as the long run. Recall for instance that with $\sigma = .40$, the probability of bankruptcy rises above one-half by $t = 10$. These dismal consequences for a no-growth policy represented by Eqs. (1) to (6) are true regardless of the values the capitalist assigns to or enjoys for the investment, consumption, profit and depreciation rates as long as they satisfy the no-growth conditions of Eq. (7).

The firm referred to in the presentation of the above model is a capitalist firm for good reason. The fundamental distinction between a capitalist and socialist firm is with regard to its control, which is exercised most simply and effectively through ownership. In a capitalist economy, the control is private, so that each firm has an owner who enjoys or suffers the firm's profit or loss. That is the objective of control. The profit, wealth etc. of each capitalist fluctuated more or less from one period to the next due to the vagaries of operating in a competitive market. In aggregate over the large number of capitalists, the aggregate values of K, M and the other variables are practically constant from one period to the next. In a socialist economy, the manager of each firm does not enjoy the firm's profit. The profits of all firms flow to the state, and the constant aggregate is distributed in some way by the central authority. The insecurity and bankruptcy in our simple competitive capitalist system is completely absent in pure simple socialist system. Its hallmark is security. Of course, this socialist system might well have real world problems of motivation and growth. We will not deal with that subject here beyond noting that actual socialist systems that were fairly close to a pure socialist system, such as Stalin's Soviet Union and Mao's China, were forced to take extraordinary measures to deal with the problems of motivation and growth. Mao tried the Great Leap Forward and the Cultural Revolution.

II: GROWTH, UNCERTAINTY AND BANKRUPTCY

Before going on to extensions of our model that make it a more realistic representation of a capitalist and then considering the consequences of pursuing growth, we will attempt to review its relation to the relevant literature. Our objective is to establish where our model departs from or advances existing theory. That takes place for the most part with respect to growth, uncertainty and bankruptcy, and these topics, bankruptcy in particular, will be emphasized in the review. The treatment of bankruptcy in mainstream (neoclassical) theory has much in common with the treatment of death in religion. We will see that bankruptcy is ignored, denied, glorified or passed over quickly in the neoclassical theory of the firm or individual.

SMITH AND MARX ON GROWTH:

As indicated by the title of his great work, Adam Smith's objective was maximizing the wealth of nations. To realize that objective, he argued that: (1) there should be specialization and exchange in markets free of government and other restrictions on free competition among capitalists, and (2) each capitalist should maximize the share of operating profit devoted to the growth of capital. Operating profit is value added less the wages of production workers, and to the extent possible, it should be devoted to the further accumulation of capital. Operating profit devoted to consumption, management, the arts, general welfare etc. should be minimized regardless of how useful such expenditures might be [Smith vol. 2, bk. 2, ch. 3]. By implication, Smith disapproved of feudal lords who devoted their entire surplus to their pleasures, war, the church and other non-productive purposes.

Marx argued that the entire surplus was expropriated from the working class, and he attacked that expropriation, while occasionally expressing admiration for the progress made possible by its use. To our knowledge, Marx was the first person to state that capitalists were subject to a growth imperative. He wrote:

> Moreover, the development of capitalist production makes it constantly necessary to keep increasing the amount of the capital laid out in a given industrial undertaking, and competition makes the immanent laws of capitalist production to be felt by each individual capitalist, as external coercive laws. It compels him to keep constantly extending his capital, in order to preserve it, but extend it he cannot, except by means of progressive accumulation.
>
> So far, therefore, as his actions are a mere function of capital--endowed as

> capital is, in his person, with consciousness and a will--his own private consumption is a robbery perpetrated on accumulation, just as in book-keeping by double entry, the private expenditure of the capitalist is placed on the debtor side of his account against his capital. [Marx 1906, p.649]

For the most part, Marx attributed the behaviour of capitalists to the nature or personality of the people who become capitalists. It was not made clear what, if anything, in their circumstances drives them to subordinate all else to the further accumulation of wealth.

Rosa Luxemburg [1968] is the Marxist whose main thesis is that capitalism is subject to a growth imperative. She relied on history to prove that capitalism has always relied on predatory relations with non-capitalist systems at home and abroad to survive. She wrote “real life has never known a self-sufficient capitalist society under the exclusive domination of the capitalist mode of production” (p. 348). Her effort at a more rigorous proof of her thesis was flawed, and that resulted in its rejection by other Marxists. See Sweezy [1942, pp. 202-207]. The most favorable treatment of Luxemburg’s thesis was by Joan Robinson in an introduction to the edition cited. See also Gordon [1987].

NEOCLASSICAL AND KEYNESIAN THEORY:

Since the middle of the nineteenth century, mainstream economic theory has been neoclassical theory, a body of knowledge devoted primarily to discovering and expounding on the utopian properties of a perfectly competitive capitalist system. Departures in the real world from this utopia, were ignored, dismissed as unimportant, or recognized as imperfections to be overcome on the way to a more perfect world. Prior to World War II, the theory of investment was based on the assumption that the future payoffs on an investment are known with certainty, or what commonly amounts to the same thing, their uncertain future values can be represented by their expected values. It could then be reasoned that the capitalist maximized the market value of wealth with the investment or disinvestment decision, and that was accomplished by equating the marginal rate of return on investment with the interest rate. The utopian properties of this investment decision were explored at great length in the development of the theory of interest and capital. No reason was found why a firm could not remain in a stationary state, and the same was true of the allocation of income between current and future consumption on the part of real persons.

The combination of the Great Depression, Keynes's reputation and his literary skills finally made underconsumption and underinvestment theories of aggregate demand respectable. Previously, the instability of actual capitalist systems was attributed mainly to monetary policy, labor or "political" events. With Keynesian economics, fiscal as well as monetary policy could be used to manage the economy. From a theoretical viewpoint, the Keynesian consumption function made private investment the critical economic variable,[7] and Keynes railed against the inadequacy of the prevailing theory of investment, [both its failure to deal with uncertainty about the future and the desire for security. See Keynes [1936, bk. IV, esp. ch. 12].

Modigliani and Miller [1958] established the conditions under which the neoclassical theory of valuation and investment under certainty holds in the presence of uncertainty and risk aversion with one small qualification. To achieve this, they implicitly assumed that the corporation is nothing more nor less than a collection of separable assets and investment opportunities. That makes it possible to value each asset or opportunity at a discount rate that is greater than the interest rate by an amount that depends solely on *its* risk. Consequently, the

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7In fact, a growth imperative on a macro level has been derived from the Keynesian consumption function. At full employment, investment is positive, and to maintain full employment investment most grow with the economy, unless government grows disproportionately. See Domar [____] and Harrod [_____].

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value of a firm is completely independent (apart from taxes and other market imperfections) of its capital structure, its dividend policy and even its investment decision, the last because the value of each investment opportunity may be captured as easily by selling it as by undertaking it. Gordon [1994, chs. 2, 4, 5 and 6] summarized the widely recognized failure of the theory to explain corporate practice. The theory was equally useless in explaining how stocks are valued and in estimating the all-important cost of equity capital. See Brigham and Gordon [1968], Malkiel and Cragg [1970], Fazzari, Hubbard and Petersen [1988] and Gordon and Gordon [199__].[8]

Markowitz [1959] arrived at the set of portfolios that are mean-variance efficient in one period returns, and Sharpe [1964] made the further assumptions needed to arrive at the Capital Asset Pricing Model for shares.[9] The separability of the shares in a portfolio made it easy for Hamada [1969] to relate the CAPM to the Modigliani-Miller theory of corporate finance. However, for the CAPM to be true, the efficient market hypothesis must also be true: the expected return on a share must be equal to the average realized returns. Unfortunately, 25 years of testing led Fama and French [_____], ________[____] and others to the conclusion that there is no evidence to support the joint hypothesis that CAPM and EMH are true. See Frankfurter

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8Gordon and Shapiro [1956] and Gordon [1962] developed an alternative theory called the dividend growth model, under which the value of a firm is the present value of its dividend expectation, and its cost of capital is its dividend yield plus its expected growth rate. The theory is widely used in the practice of finance and it is widely taught in business school finance courses. It has been banished, however, from the theoretical literature on finance, probably because it recognizes that growth is risky.

9No portfolio with a higher mean return has a lower variance, and no portfolio with a lower variance has a higher mean return. With lending and borrowing at a risk-free interest rate, the set of the efficient portfolios is combinations of the market portfolio and the risk-free asset. The expected one-period return on the jth share is then $\mu_j = r + \beta_j (\mu_m - r)$, where r = the risk-free interest rate, μ_m = expected one-period return on the market portfolio, and β_j = systematic risk of the jth share.

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[______].

Papers by Samuelson [1969] and Merton [1969] [1971]established the consumption and portfolio policy (allocation of wealth between a risk-free asset and a diversified portfolio of risky shares) over time of an individual whose objective is the maximization of *expected* utility of future consumption. The individual is assumed to have constant relative risk aversion, that is, the allocation is independent of the level of wealth. The fraction of net worth invested in the risky share portfolio is found to be

$$\pi = (\mu - r) / \beta\sigma^2 \qquad (8)$$

with μ = mean rate of return on the portfolio, r = risk-free interest rate, β = investor's degree of relative risk aversion and σ^2 = variance of the portfolio's rate of return. On average over all investors, π should be equal to one in a closed system without government and π should be below one if investors own but do not owe the public debt. The data are consistent with this conclusion, since $\mu - r$ is about .06, and σ^2 is about .045. An investor who puts all wealth in the risky portfolio has $\pi = 1$, and that implies $\beta = 1.33$, which is a reasonable degree of risk aversion. As β and σ^2 rise above these values or the risk premium $\mu - r$ falls below .06, π falls below one.[10]

10 Robert Lucas and his followers have developed a macroeconomic model in which the risk of holding the market portfolio is the *variance in the growth rate of per capita consumption*, and that has given rise to the "equity premium puzzle." See Mehra and Prescott [1985] and Kocherlakota [1996]. The latter reports on p. 47 that the variance in the per capita consumption growth rate is about .0013. That figure, combined with an equity premium of about .06 would require a degree of relative risk aversion that is well beyond all reason, if we are to have $\pi = 1$ in Eq. (8). We do not understand the measure of risk used in the Lucas model, unless the model is intended to represent no more than a pure exchange economy with the sole asset an *endowment* of grain that is consumed, stored or loaned, and with the mean return on storage positive. In an economy that employs land to produce grain, land would be the dominant asset; its value would be the discounted value of its expected future output, and the variance in its return would be the change in the discounted value of that expectation from one period to the next. Its risk would be far greater than the variance in the growth rate in grain consumption.

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BANKRUPTCY:

Samuelson was disappointed to find that, contrary to popular belief, the model found that the fraction of wealth consumed and the allocation of wealth between the two assets were independent of the age, wealth, and other properties of the individual. Constant relative risk aversion makes π independent of wealth on the micro level. Troubled by the fact that there was no explicit recognition of the possibility of bankruptcy in the Samuelson-Merton model, Suresh Sethi explored the extension of the model, to incorporate bankruptcy. What resulted was a series of papers that were collected in Sethi [1997], where a paper by Gordon and Sethi [1997] provided an economic interpretation of this work.

The failure to explicitly recognize bankruptcy in Samuelson and Merton did not make it a non-event. Paradoxically, the failure made it an infinitely terrible event, so terrible that it is never allowed to take place. In their model, a peasant whose wealth was reduced to one kilo or even *one grain* of rice would divide it into three parts: s/he would consume a fraction, plant a fraction and store a fraction in case the crop failed. These fractions would be determined without regard for the fact that the peasant could not and would not even attempt to survive on the amount consumed. Gordon and Sethi [1997] [1998], recognized that there is life after bankruptcy by having the individual maximize the sum of P at the end of t periods, and the *expected* utility of consumption over the t periods, where t is the uncertain date at which bankruptcy will take place, and P is the utility of consumption subsequent to bankruptcy. At t, the individual enters a new state, becomes a beggar, a thief, takes a job, etc., and the income to be earned thereafter in that state has a utility of P.

This extension of the Samuelson-Merton model produces intuitively attractive results.

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Notwithstanding the assumption of constant relative risk aversion, consumption expressed as a fraction of wealth and the fraction of wealth invested in the risky asset both decline as wealth increases. This investment policy means that as wealth falls toward zero, the individual adopts a go-for-broke investment policy. This policy is to borrow and invest to the limit allowed by creditors, since a conservative financial policy makes bankruptcy certain within a few periods, and the preferred alternative is a high probability of bankruptcy in the nearer future, along with some chance of repairing one's fortune. The fraction of wealth consumed also rises as wealth falls toward zero, for the simple reason that consumption will certainly not be allowed to fall below what it would be in the event that bankruptcy takes place. Corporations can be expected to behave similarly with respect to their investment and overhead expenditures.

The theoretical literature that recognizes the existence of bankruptcy also has treated it in a curious way. Two theories of bankruptcy exist: the absolute priority rule and the relative priority rule. Under the former, bankruptcy takes place when the value of the firm falls to the amount of its debt, in which case the corporation is reorganized with all creditors paid in full and the equity holders left with nothing. Neoclassical economists argue that the absolute priority rule should be followed, claiming that a bankruptcy would then be a relatively costless and unimportant event, like the refinancing of an outstanding debt issue by a strong corporation. See, for example, Senbet and Seward [1995]. An implicit assumption in the advocacy of the absolute priority rule is that an exact determination of a firm's market value is a trivial task.

In fact, the question of bankruptcy does not arise with a determination that the market value of a corporation's assets has fallen below its outstanding debt. The question arises when a corporation violates one of its debt contracts and the creditor demands payment. What the law

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provides for in that event is the basis for the historical development of the relative priority rule. In order to insure an orderly disposition of the firm's assets, the corporation may and usually does obtain the protection of the courts when a creditor demands payment because the corporation has violated a covenant of the debt contract. The court then may and usually does arrange a reorganization of the corporation in which the claims of the investors in the corporation (usually all of them, including stockholders) are recognized on the basis of their priority, the weakest claims being scaled down the most. Practically all business bankruptcies take place under the relative priority rule, because the law makes it available to corporations. The costs of bankruptcy can be substantial, according to neoclassical economists, onlybecause the misguided intervention of government has resulted in legislation that enshrines the relative priority rule, and it results in litigation, uncertainty and other costs. Otherwise, bankruptcy would be a non-event.[11]

It may be wondered why we use the book instead of the market value of capital in deciding when M/K puts the firm in bankruptcy. Although the universal use of market value in decisions about the firm is advocated in some quarters, it is only useful for some purposes, and bankruptcy is not one of them. M/K at market is far more volatile than a cost based ratio, since the market-to-book ratio for the value of capital can fall below minus one as well as rise above one, with the former taking place as bankruptcy approaches. Furthermore, it is quite impossible for a firm to maintain some debt-capital ratio at market value, and fairly possible to do so at book value. Regardless of what triggers bankruptcy, its likelihood increases as the debt-capital ratio at

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11 For the development of this reasoning, see Senbet and Seward [1995], Baird [1996] and numerous other essays in Bhandari and Weiss [1996]. We could not find a defense of the relative priority rule other than Warren [1996], who argued that its widespread use makes it deserving of consideration.

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cost rises, and when it does take place, so little is given to the old owners in a reorganization that we may take it to be zero. The assumption here that bankruptcy takes place when the ratio rises to .5 is arbitrary but reasonable. In addition, raising the ratio to .75 or even .9 would have no theoretical or material impact on the conclusions reached. In an economy without bankruptcy laws that protect firms against their creditors, it would still be triggered by the failure of the firm to meet the obligations of its debt contract, and we assume that takes place when the debt-capital ratio rises to .5. At that time, the debt-equity ratio at market value would likely be higher, and possibly over one.

It may be argued, bankruptcy is of no concern on the level of a market or an entire economy, because we have birth as well as death: new firms replace old firms and life goes on. However, the knowledge that they will be replaced rarely if ever makes death or bankruptcy of no concern for real and corporate persons. What they do to avoid or delay their ultmate fate is what makes death or bankruptcy important for understanding the behaviour of both real and corporate persons.

III. EXTENSIONS OF THE MODEL

The model represented by the simulation in Table I is unrealistic in a number of ways. For instance, the closed competitive capitalist system it is intended to represent has existed only in theory: its use may be justified only as a starting point for the consideration of more realistic models. What we will do now is explain or improve on what we consider to be the most glaring or objectionable of the assumptions incorporated in the model.

DEPENDENCE ON PROFITABILITY

Perhaps the most obvious objection to the plausibility of the model represented in Table I

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is the assumption that consumption and investment are independent of income or profitability. They both move only with K, and it changes little from one period to the next. In fact, a policy of making investment equal to depreciation makes K remain constant at $1,000 over time for the surviving capitalists. Clearly consumption and investment should in part respond to income or profitability, and changing our model to realize that objective is accomplished by replacing Eq. (5) with:

$$C(t) = \eta K(t) + \max[y SP(t) \text{ or zero}], \tag{5A}$$

and by replacing Eq. (6) with:

$$I(t) = g\lambda K(t) + \max [h SP(t) \text{ or zero}]. \tag{6A}$$

SP(t), the smoothed profit, is an exponential average of its previous value and current profit.

$$SP(t+1) = \gamma SP(t) + (1 - \gamma) P(t), \tag{9}$$

and the initial value of SP is $SP(1) = \mu K(1)$.

Eq. (5A) is a Keynesian consumption function. It makes the reasonable statement that in each period there is a "minimum" consumption. It is represented by $\eta K(t)$, so that the minimum changes slowly over time with capital valued at cost. In addition, consumption varies with

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expected income, but C(t) cannot fall below ηK(t), even though SP(t) < 0 is possible.[12] Eq. (6A) states that with g < 1 there is a minimum expenditure on asset replacement regardless of profitability that is less than λK. The actual level of investment depends also on profitability, but this component of investment cannot be negative, so that gross investment cannot fall below gλK. Finally, the senstivity of C, I and SP to current profit increases as γ falls toward zero.

Is a no-growth policy violated by making C and I partially dependent on profitability? No. We now replace the conditions q = gλ + η = μ and g = 1 with the conditions that the expenditure rate and the investment rate,

$$q = g\lambda + \eta + \mu(y + h) = \mu \quad \text{and} \quad ivr = g\lambda + \mu h = \lambda. \tag{10}$$

With these conditions satisfied, and with 0 < or = γ < or = 1, K and M, as well as the decision variables, remain in a stationary state when μ is certain, which is our condition for a no-growth policy.

Our appendix proves that when r = 0 and either the “max” constraints are removed from Eqs. (5A) and (6A) (Theorem 5), or γ = 1 (Theorem 6) or γ is near 1 (Theorem 7), the above model satisfies a martingale property, which means that the firm is certain to go bankrupt in the long run. The proof is difficult if not impossible under the max conditions of Eqs. (5A) and (6A), but simulations with and without the max conditions under a wide range of circumstances revealed that for every combination of σ and t, the probability of bankruptcy was greater with the max conditions than without them. We therefore can say with great confidence that bankruptcy is certain in the long run for the system represented by the above model.[13]

12A corporation does not consume and it is not obliged to pay a dividend, but the overhead cost of managing the corporation and the expenditures in pursuit of monopoly power do not go to zero as operating profit goes to zero. See Gordon [1998].

13 The expected values of K and M conditional on survival were about the same or higher with

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To see what happens in finite time, we simulated the model under a range of values for the parameters that seemed of interest. Of particular interest is what happens to the probability of bankruptcy as the dependence of C and I on profitability is raised by moving γ from $1 \div 0$ and by raising $y + h$ from $0 \div 1$, the latter with compensating reductions in $g\lambda$ and η to satisfy the conditions that $q = \mu$ and $\lambda = g\lambda + \mu h$. Either when $\gamma = 1$ or when $y = h = 0$ is true, there is no dependence of C and I on profitability, and we are back with the model of Section I that produced the output in Table I. When $\gamma = 0$ and $y + h = 1$ with $\eta = g\lambda = 0$, there is complete dependence on profitability, since $\gamma = 0$ makes SP equal to current profits, and the minimum levels of both C and I are zero. The simulations revealed that the probabilities of bankruptcy in finite time are substantially unchanged.

Table II presents the simulation results when C and I are made dependent on both capital and profitability by setting $\gamma = .5$, with g and η made positive and h and y taking the values needed to satisfy Eq. (10). We kept $\mu = .25$ and $\lambda = .10$, and the values of g, h, η and y were set to arrive at a reasonable distribution between the "fixed" and "variable" components of C and I, while keeping the expenditure rate $q = \mu$ and the investment rate equal to the depreciation rate. Comparing the statistics in Tables I and II reveal that partial dependence of C and I on profitability provides less attractive prospects than no dependence.

We simulated the profitability model over a wide range of values for the above parameters without violating the conditions in Eqs. (10) that the expenditure rate is equal to the profit rate when the profit rate is equal to its mean, and the investment rate is equal to the depreciation rate. In particular, we tried μ above and below the .25, raised and lowered the

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the max condition.

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relative importance of C and I, and raised or lowered the fixed and variable components of C and I. Unlike the model of Section I, where the probability of bankruptcy and other statistics were independent of these parameter values, here the statistics change in one way or another. However, as long as the no-growth conditions of Eq. (10) are satisfied and $r = 0$, the probability of bankruptcy for each combination of t and σ varied over a limited range from one simulation to the next. The values of K and M, conditional on surviving for t periods, varied over a fairly modest range, except when the probability of bankruptcy is large, in which case the values of K and M are for a small sample.

Our conclusion is that the extension of our no-growth policy to make expenditure partially dependent on profitability does not eliminate the unattractive features of the policy. The opposite is true. Bankruptcy is still inevitable in the long run. Furthermore, the probability of bankruptcy in finite time is commonly raised for each value of t, and the unconditional expected value of the growth in wealth is still negative. The lack of growth in a no-growth policy is still not compensated for by a high probability of survival.

OTHER REFINEMENTS:

The assumption that the interest rate $r = 0$ was not made solely for convenience, in that the theorems proved in the Appendix either could not have been proven, or would require a far more complicated argument. However, recognizing that interest is paid on loans has a negligible impact on the survival probabilities and on the expected value of K and M under a no-growth policy. We established this by simulating the model represented in Table II with reasonable values assigned to the interest rates on short-term and long-term debt and a reasonable structure to the components of M. Of course, a very large initial wealth, a very high rate of interest on

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money loaned, very low values for C/K and I/K, or a string of exceptionally profitable rates of return on K, might make it possible and attractive for a capitalist to become a rentier and live forever on M. However, these conditions are exceptional, and need not concern us.[14]

Another possible objection to the models represented by Tables I and II is the growth in M/K over time, conditional on the firm's survival for the t periods. M grows beyond all reason in relation to K in Table I, and the growth in M/K is still large in Table II, where excess profits are absorbed in part by C and I. M = 0 is a strong financial position, and having M > K, as takes place in these simulations, provides exceptional security. It is quite possible that any particular capitalist is so obsessed with the fear of bankruptcy that all excess cash is held as money, but that is exceptional behaviour. There are two polar alternatives for capitalist policy. One is to spend all excess M on additional consumption, and the other is to devote it to the further accumulation of capital. Regardless of whether the excess cash is put into C or I, it can be argued that the firm is following a no-growth policy, since all the variables, C, I, K and M, would remain constant over time if only μ could be realized with certainty. Under both policies, the expected value of M is kept within reason, but the probability of bankruptcy rises more sharply with σ and t. Hence, putting excess cash into some combination of consumption and capital does not improve the prospects for survival. Quite the contrary, they are reduced materially, the only compensation being that, conditional on surviving for t years, the intervening years are somewhat more satisfying.

It might be wondered why we did not include inventory in M or include it in the model as

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14 In the real world, we have rentiers who live solely on interest income, but that is possible only through a redistribution of income through government and business debt. Income is produced by K and not by M.

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an asset separate from M and K, since inventory, unlike other real assets, is sold for money in the course of business operations. However, the very high degree of specialization and exchange in a modern capitalist economy severely limits the liquidity of the inventories held for production or distribution. The inventory is held for profit in the same way as machinery, buildings and land, because the cost of storage or lack of it rises sharply as the inventory departs in either direction from an optimal inventory output ratio. The inventory that is readily liquidated to meet the demand for cash is held by speculators who take a financial position in it in order to profit from the change in its price. Speculation on the change in commodity prices is relied upon to provide security of supply. Since the inventory that can be liquidated to meet the need for cash is very small in relation to other non-monetary assets, M/K as defined here may be used to determine the level of security.

SERIAL CORRELATION AND MEAN REVERSION

An interesting and reasonable departure from our assumption that the expected rate of profit on capital is μ regardless of its realized value, is the assumption that realized values of α(t) convey information about future values of μ(t). This dependence of the expected return on capital on its realized values and a tendency for the expected return to revert towards some long run value is captured by the expression:

$$\mu(t+1) = c(1)\mu(t) + c(2)\alpha(t) + (1 - c(1) - c(2))\mu(1) \qquad (11)$$

with the three coefficients all positive and their sum equal to one. The change in the expected return on capital and investment that now takes place due to their dependence on realized returns makes the market value of K depart from its cost value. That is no problem, since market value does not enter our model, as explained in the previous section. Bankruptcy in the long run may

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not be certain under the serial correlation and mean reversion that we have in Eq. (11). See the Remark just before Theorem 6 in the Appendix.

What happens in finite time would seem to depend materially on the relative values of the three coefficients of Eq. (11), and we simulated the model described in Table II with a limited range of values for the coefficients of Eq. (11). Table III presents the simulation results with $\mu(1) = .25$, $c(1) = .5$ and $c(2) = .4$, and with the other parameters the same as in Table II. As can be seen in Table III, serial correlation with mean reversion still has the probability of bankruptcy increase with σ and t. Comparison of the data in Tables II and III reveals that in Table III the probability of bankruptcy rises much more rapidly with t for low values of σ and somewhat less rapidly for high values. What seems to be true is that with serial correlation, a low value of σ no longer confers a low probability of bankruptcy. Another pronounced difference between Tables II and III is the rates of growth in K and M, conditional on survival for t periods. They are much much greater with serial correlation, which is what one would expect. Even the unconditional values of K and M rise with time, once some critical point is passed. However, the probability of bankruptcy continues to rise with t for all values of σ, suggesting that bankruptcy is only a matter of time unless the no-growth policy is abandoned. On the other hand, serial correlation may only make bankruptcy highly likely but not certain in the long run.[15]

We repeated the simulation in Table III with a few other combinations of the coefficients of Eq. (11). For $c(1) = .4$ and $c(2) = .5$, for $c(1) = .6$ and $c(2) = .3$, and for $c(1) = .4$ and $c(2) = .3$, the rates of growth in the probability of bankruptcy with σ and t were remarkably close to the

15 That certainly is true if a more conservative consumption and investment policy is adopted. On the other hand, with great wealth, non-economic considerations become increasingly important and costly in the activities needed to protect the wealth.

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values in Table III. On the other hand, the rates of growth in K and M were raised, particularly for high values of σ and t, as c(2) was raised relative to c(1). The rates of growth in K and M were reduced materially as the mean reversion was raised by reducing the sum of c(1) and c(2). We did not carry out simulations for radically different relative values for the coefficients of Eq. (11), or for other values of the other parameters in Table III.

It is possible that the simulation results with dependence and mean reversion are influenced by the value of the random draw at which α(t) is started more than takes place without the dependence {TO BE CONTINUED}

IV. GROWTH POLICIES

What might a firm do to reduce materially the high probability of bankruptcy that it faces under the no-growth policies examined above? One alternative to a no-growth policy is to raise the profit rate above the expenditure rate, that is, make $q < \mu$ in Eq. (10), while keeping the investment rate equal to the depreciation rate. Another is to make the investment rate greater than the depreciation rate, while keeping the expenditure rate equal to the profit rate. A third is to adopt both departures. We cannot say that bankruptcy is certain in the long run under any of these alternatives to a no-growth policy, and we simulated all three. Raising the profit rate above the expenditure rate with the investment rate left equal to the depreciation rate improved the survival rate materially by comparison with Table IV, particularly for low values of σ. However, the growth rates in C, I, and K, conditional on survival, were reduced even closer to zero. Raising the investment rate resulted in materially higher growth rates in K and M and in C, but these growth rates over t periods were conditional on survival for the t periods, and the

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probability of survival was not improved.

Table VII presents the simulation results when both departures from the no-growth policy represented in Table IV are adopted and the departures are quite large. There, μ is raised to .30, and the investment rate is raised to .15, with q = .25 and λ = .10 maintained by compensating changes in the other parameters of the simulation presented in Table IV. The survival and growth statistics in Table VII realize the best of both departures from a no-growth policy. Although we cannot state categorically what will happen in the long run, it would seem likely that for departures from a no-growth policy as great as those in Table VII, bankruptcy in the long run seems quite unlikely for $\sigma < .2$. What happens in finite time is quite interesting by comparison with Table IV. The probability of bankruptcy remains at or close to zero up to t = 100 for $\sigma = .20$. The probability rises sharply with t at $\sigma = .40$, but for each value of t, it is still substantially lower than in Table IV. At $\sigma = .60$, the probability of bankruptcy for all values of t is not much lower than under the no-growth policy of Table IV. The striking departure in Table VII from Table IV is in the high rates of growth in K and M, their unconditional as well as their conditional values. Clearly, the growth policy represented in Table VII offers far more attractive prospects than the no-growth policies examined in the previous section.

There are, however, two problems with this growth policy. Any one capitalist can follow a policy of making the expenditure rate less than the profit rate, but that policy cannot be followed widely in a closed capitalist system without government. In aggregate, we must have the expenditure rate equal to the gross profit rate, unless there is lending to, or investment in, other sectors -- government, production workers or the foreign sector. The other problem with the policy is that it is Keynesian economics: with the limited exception of loans to government to

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finance military expenditures, loans as well as gifts to others beyond charity are not popular with capitalists. The decline in Keynesian economics over the last quarter century reflects the success of capitalists in the advanced capitalist countries in pursuing a different growth path, one that had been pursued with increasing vigour and success over the whole century.

On the macroeconomic level, these policies have come to be represented by what is called "endogenous" growth theory. In contrast with traditional neoclassical theory, where the source of productivity growth is "exogenous" to the system, here the system generates the growth in human capital and other sources of technological progress that are the bases for the productivity growth that endogenous growth theory seeks to explain. See Solow [1970], Roemer [1994] and Nelson [1997]. On the microeconomic level, the neoclassical theory of the firm maintains that production is the only activity in which the firm engages; labor and capital are employed solely for that purpose.

In fact, firms engage in a wide range of non-production activities in the pursuit of the monopoly power that is needed to make the production profitable.[16] The activities undertaken to maintain and increase monopoly power range from innovations in technology, marketing and management, to influencing government in various ways, including economic, political and military relations with third world countries. These countries also contribute to the gross profit on production through the terms of trade and foreign investment. See Barnet and Cavanagh [1994], Greider [1997], and Thurow [2000]. An increasing fraction of the personnel of firms are

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16 The distinction between production and non-production employees has a long and distinguished history in economics. Smith [1822: Vol. II, Book II] used the distinction in his policies for maximizing the wealth of nations and Marx used the distinction in his theory of surplus value. Since at least 1846, the U.S. Census of Manufactures reports the employment and compensation of all employees and the corresponding figures for production workers.

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employed to increase the gross profit on production, the latter rises relative to the cost of production, and an increasing fraction of the gross profit finances employing people in the activities that increase the gross profit. The ratio of output (value added) to the compensation of the workers employed in its production is a measure of the monopoly power a firm enjoys. See Kalecki [1954] and Gordon [1998]. The latter found that the degree of monopoly power in the U.S. manufacturing sector fluctuated in a very narrow range, around 2.50 between 1899 and 1949, and then it rose sharply to 5.25 between 1949 and 1994.

Abandoning the assumptions that the system is closed and the firm is one of a large number of buyers and sellers in competitive markets opens up other attractive possibilties for raising both the growth and survival rates. Monopoly power is a means for raising the mean and reducing the variance of the return on capital, thereby raising both the growth and survival rates. Diversification over different product lines or different industries, particularly through the mergers or acquisitions made possible by the corporate form of organization, reduces the variance of the return on capital.

Our model is readily extended to capture the expenditures in the pursuit of monopoly power that increase profitability in various ways including productivity growth. Recall that the gross profit on production is devoted to both investment and consumption, with the latter including the managerial and professional personnel who are employed to increase the gross profit. Growth through monopoly power is captured by having the gross profit increase at some annual rate, while capital, K, grows at a modest rate due to investment. Growth in the profit rate is devoted largely to compensation of the personnel employed for that purpose, but it may also raise the net rate of return on capital and the ratio of market-to-book value for K.

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Table VIII presents the simulation results when μ grows by one percent of its value annually, ivr = .12 > λ = .10, and the parameters η and λ grow at the same rate as μ to keep the expenditure rate equal to the profit rate. By comparison with Table IV, we see that the survival rate is improved materially for low values of σ, but not for σ = .40 or larger. Hence, expenditures in the pursuit of monopoly power increase the probability of survival only if risk is reduced by mergers, acquisitions and other activities. The growth in K and M, both conditional on survival and the unconditional values compare very favorably with their growth in Table IV. Of course, as σ and t rise, the unconditional growth in K and M falls relative to the growth with σ = 0. Higher values for the investment rate and the growth in the profit rate increase the survival rate somewhat and raise growth rates in K and M materially.

Table IX reveals that more attractive growth in K and M is achieved when the positive net investment and one per cent per period growth in the gross profit rate in Table VIII is combined with the serial correlation and mean reversion coefficients in Table VI. The unconditional values of W = K + M never turn downward; they soon pass the values when σ = 0, and they are quite attractive for combinations of σ and t that are large. This remarkable growth takes place with q = μ maintained, and with net investment only 2 per cent of capital stock in each period. On the other hand, the probability of bankruptcy rises with t for low values of σ even more rapidly here than in Table VI, and far more rapidly than in Table VIII. Raising the growth rate in μ to 1.5 per cent per period had practically no impact on the statistics, but doubling the net investment rate to 4 per cent of K raised the growth rate in K + M enormously. Our model, in which there is growth in q = μ, positive net investment, and serial correlation with mean reversion captures important stylized facts about the behaviour of U.S. firms. Further

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exploration of the model on the bases of plausible ranges of values for the parameters may prove worthwhile.

V. CONCLUSION

The previous pages have established that under simple but realistic models of a firm operating in a closed competitive capitalist system, a no-growth policy makes bankruptcy certain in the long run and probable to an intolerably high degree in finite time. Furthermore, the expected value of the growth rate in wealth is negative. We found this to be true with consumption and investment dependent on any combination of the capital stock and profitability, as long as the expected return on capital is independent of its realized values. We could not establish whether or not bankruptcy remains certain in the long run when serial correlation and mean reversion are introduced into the expected return. The probability of bankruptcy in finite time is not far different from its values when the profit rate is independent of its history, but the prospects of the firm become considerably more variable, and the growth rates in K are much greater with dependence.

Policies that pursue growth improve the firm's prospects greatly. It can be pursued in various ways. What may be called "Keynesian" growth policies involve both a positive net investment and an expenditure rate that is less than the expected profit rate. They improve the growth rate in wealth materially, and raise the survival rate somewhat. However, they involve investment or lending to government, to workers, or abroad, and doing so is not popular with capitalists. More attractive and widely adopted are policies which involve expenditure in the pursuit of monopoly power and the growth in gross profit to finance these expenditures. Positive

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net investment combined with growth in the expected value of the gross profit rate and in the expenditure rate created the possibility of substantial improvement in the growth rate in wealth, but the survival rate is not improved. In fact, the problem of survival may become more challenging.

APPENDIX

In this Appendix, we prove various results claimed in the text.

We begin with a general result which says that our model only depends on seven particular combinations of parameter values.

THEOREM 1:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). The distributions of K(t) and M(t), for all t>=1, depend on the various parameter values only through the seven quantities sigma, r, h, y, gamma, (g-1)*lambda, and mu-eta-g*lambda. That is, if the individual parameter values are changed in such a way that these six quantities are unchanged, then the distribution of K(t) and M(t) is unchanged for all t>1.

PROOF:

We can write alpha(t) = mu + sigma*N(t), where N(t) are standard normal random variables (i.e. with mean 0 and variance 1).

In terms of this, we compute algebraically that

K(t+1) = K(t) [1+(g-1) lambda] + h max[SP(t),0] (A1)

and

M(t+1) = M(t) [1+r] + K(t) [sigma*N(t) + mu-eta-g*lambda] (A2)
- (y+h) max[SP(t),0]

We thus see explicitly that these equations can be written entirely in terms of the six stated quantities, which completes the proof. [#]

We now turn to issues of bankruptcy. We require some preliminaries.

We shall have occasion to consider continuous-time versions of the processes K(t) and M(t), denoted KK(s) and MM(s), respectively. To define them, fix all the model parameter values and an integer t >=1. Then, given values of K(t) and M(t), find the (deterministic) value K(t+1) and the distribution of M(t+1) of the form Normal(m,v), both specified by our model. In terms of all of this, we define KK(s) and MM(s) for t <= s <= t+1, in differential form, by

KK(t) = K(t);
MM(t)=M(t);
d KK(s) = (K(t+1)-K(t)) ds;
d MM(s) = (m-M(t)) ds + Sqrt[v] dB_s;

here {B_s} is standard one-dimensional Brownian motion.

These continuous-time processes KK(s) and MM(s) have been defined precisely so that KK(t)=K(t) and MM(t)=M(t) for every integer t. Indeed, these processes simply EXTEND our discrete-time processes to continuous time. (In the language of stochastic processes, we have EMBEDDED our discrete-time processes into continuous-time processes.)

Using this continuous-time embedding, we give a precise definition of "bankruptcy" as we shall prove it.

DEFINITION: We shall say that "bankruptcy is certain" for a model, if the following holds: With probability 1, either KK(s)+MM(s) is negative for some s>1, or else KK(s) (and hence also K(t)) converges to 0 as s (and t) go to infinity.

In terms of this definition, we have the following.

THEOREM 2:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). Suppose that the model parameters are such that for all t>=1, the conditional expected values of K(t) and M(t) satisfy that

$$E[\ (K(t+1)+M(t+1)) - (K(t)+M(t))\ |\ K(t),M(t)] <= 0. \qquad (A3)$$

Suppose further that r=0 and sigma>0. Then bankruptcy is certain (regardless of the initial values K(1) and M(1)).

PROOF:

Let KK(s) and MM(s) be the continuous-time versions of K(t) and M(t), as above. Let

$$tau = inf\{\ s>1;\ KK(s)+MM(s) <= -1\ \}.$$

Define a new process {X_s} by

$$X\_s = \begin{cases} KK(s)+MM(s), & tau < s \\ -1, & tau >= s \end{cases}$$

(In particular, note that X_t = K(t)+M(t), for any integer t>=1.) That is, X_s is the total wealth of the player at time s, except that {X_s} gets "stopped" at -1 as soon as the player's total wealth hits this value.

In terms of {X_t}, equation (A3) says that E[X_{t+1}-X_t | K(t),M(t)] <= 0. Because of the construction of {KK(s)} and {MM(s)}, it follows from this that E{X_{s+r}-X_s | X_u (u <= s)]

<= 0 for any r>0. That is, the process {X_s} is a SUPERMARTINGALE, i.e. on average it will stay constant or get smaller.

In particular, E[X_s] <= X_1 = K(1)+M(1) for all s. On the other hand, since {X_s} was "stopped" as soon as it hits -1, we see that X_s >= -1 for all s>1.

We conclude that {1+X_s} is a nonnegative supermartingale with bounded expectation. It then follows from the standard Martingale Convergence Theorem (see e.g. Theorem 14.2.1 of Rosenthal [2000]) that with probability 1, the sequence {X_t} must converge pointwise to some limiting random variable, say X.

Now, if X = -1, then in particular X_s<0 for some s. This implies that MM(s)+KK(s) < 0. Hence, in this case the player was bankrupt before time s.

On the other hand, if X > -1, then {MM(s)+KK(s)} converges to X, hence also {M(t)+K(t)} converges to X. In this case, (K_{t+1}+M_{t+1}) - (K_t +M_t) converges to 0. But from equation (A2) above, we see that conditional on K_t and M_t, the difference (K_{t+1}+M_{t+1}) - (K_t+M_t) has conditional variance (sigma*K(t))^2. If the difference converges to 0, then this variance (sigma*K(t))^2 must converge to 0. Since we are assuming that sigma>0, then this implies that K(t) converges to 0. Hence, if X > -1 then K(t) converges to 0.

Since we always have either X > -1 or X = -1, this completes the proof of the theorem.   [#]

THEOREM 3:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). Suppose that r=0, sigma>0, and that mu <= lambda + eta. Then bankruptcy is certain (regardless of K(1) and M(1), and regardless of the values of g, h, eta, y, and gamma).

PROOF:

Again let X_t = K(t)+M(t) be the total wealth of a given player. Then

```
X_{t+1} - X_t     = -lambda*K(t) + alpha(t)*K(t) - eta*K(t) - y*max[0,SP[t]]
                  = (alpha(t)-lambda-eta)*K(t) - y*max[0,SP[t]]
```

Recall now that alpha(t) has mean mu, so that E[alpha(t)-lambda-eta] = mu-lambda-eta <= 0. Furthermore max[0,SP[t]] >= 0. Therefore,

E[X_{t+1} - X_t | K(t),M(t)] <= 0,

i.e. {X_t} is a supermartingale.

The result now follows from Theorem 2. [#]

REMARK: Note that this Theorem remains valid for any value BETA <= 1, not just BETA=0.5.

REMARK: Suppose mu = g*lambda + eta. In this case, if g<=1, then the conditions of Theorem 3 are satisfied and eventual bankruptcy is certain. However, if g>1, then the conditions of the Theorem are not satisfied, and it may be possible to never go bankrupt, or at least to postpone bankruptcy longer.

REMARK: This theorem can be interpreted informally as saying that, if the parameters satisfy the given conditions, then bankruptcy is certain in the long run. Strictly speaking, our conclusion of bankruptcy must also allow for the possibility that a player will keep all their wealth in cash M(t)>0 even as their capital wealth K(t) goes to 0. However, this possibility can be ruled out of g >= 1, as the following Theorem shows.

THEOREM 4:

If the hypotheses of Theorem 3 are satisfied, and if furthermore g>=1, then there is s>1 with MM(s) < -BETA*KK(s) for any BETA <= 1.

PROOF:

If g>=1 then K(t) >= K(1) for all t >= 1. It is therefore not possible that K(t) converges to 0 in this case. According to Theorem 3, the only other possibility is that there is s>1 with MM(s) +KK(s) < 0. The result follows. [#]

We now consider further the "no growth condition" discussed in the text, namely

(g-1)*lambda + h*mu = mu-eta-g*lambda-mu*(y+h) = 0.   (10)

If y=h=0 then this equation implies that mu = g*lambda + eta; hence, if g <= 1 then the conditions of Theorem 3 are satisfied. However, if y and/or h are positive the situation is less clear. We have the following.

THEOREM 5:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). Suppose we replace max[SP(t),0] with SP(t) in Eqs. (5A) and (6A), leaving equations (1) through (4) and (9) unchanged.
Suppose further that (10) holds and that r=0. Then for this modified model, E[K(t)]=K(1) and E[M(t)]=M(1) for all t>1 (i.e., on average there is zero growth). Hence, if sigma>0, then Theorem 2 applies, and bankruptcy is certain.

PROOF:

We prove by induction that E[K(t)]=K(1) and E[SP(t)]=mu*K(1). Recall that SP(1)=mu*K(1), hence the statement is obviously true for t=1.
Now assume it is true for some t >= 1. Then from equation (A1) above, with max[SP(t),0] replaced by SP(t), since (g-1)*lambda + h*mu = 0 and E[SP(t)]=mu*K(1), we see that E[K(t+1)]=K(1). Now, since P(t) = alpha(t)*K(t), and since alpha(t) and K(t) are independent, it follows that E[P(t)]=mu*E[K(t)]=mu*K(1). But then since

SP(t+1) = gamma*SP(t) + (1-gamma)*P(t),

it follows that E[SP(t+1)] = mu*K(1). Hence, it follows by induction that E[K(t)]=K(1) and E[SP(t)]=mu*K(1) for all t>1.

But once we know that E[SP(t)]=mu*K(1) and (10) holds, then it follows from equation (A2) that E[M(t+1)]=E[M(t)] for all t>=1. Hence, by induction again, E[M(t)]=M(1) for all t>1. This completes the proof. [#]

We conclude from Theorem 5 that, if (10) holds, then any possible escape from eventual bankruptcy must occur entirely due to the difference between max[SP(t),0] and SP(t) in our model.

REMARK: If we do not replace max[SP(t),0] with SP(t) in Eqs. (5A) and (6A), then Theorem 5 does not apply. However, it is not clear in this case that the probability of bankruptcy is reduced. Indeed, the simulations presented in the text indicate that eventual bankruptcy is certain in this case as well. (Mathematically speaking, the "max" in Eqs. (5A) and (6A) slightly increases both I and C. The increase in C can only lower the player's wealth. However, the increase in I is more subtle, which prevents a clear mathematical analysis.)

REMARK: Theorem 5 assumes (as do all other results in the appendix) that the mean profit rate, mu, is held constant. If we instead assume mean reversion for mu, as in equation (12) of the text, then alpha(t) and K(t) are no longer independent. Hence, Theorem 5 does not apply in this case.

THEOREM 6:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). If (10) holds and r=0 and mu >= 0 and gamma=1, then E[K(t)]=K(1) and E[M(t)]=M(1) for all t>1. Hence again, if sigma>0, then bankruptcy is certain.

PROOF:

In this case, SP(t) = mu*K(1) > 0 for all t, so that max[SP(t),0] = SP(t), and Theorem 5 applies directly. This completes the proof. [#]

THEOREM 7:

Consider the model presented in the text as equations (1) through (4), (5A) and (6A), and (9). If (10) holds and r=0 and mu>0, then for fixed sigma>0 and fixed epsilon>0, the probability that the player goes bankrupt or inf_t K(t) is less than epsilon goes to 1 as gamma goes to 1. In symbols,

lim P[bankrupt, or inf_t K(t) < epsilon] = 1. gamma -> 1

PROOF:

As gamma goes to 1, the variance of SP(t) goes to 0. But SP(t) has positive mean. Hence, max[SP(t),0] becomes a closer and closer approximation to SP(t). Consequently, the no-growth conditions of Theorem 5 get closer and closer to being satisfied.

Now, write p(gamma,epsilon,t) for the probability that, for a given gamma, the player goes bankrupt by time t, or K(t) is less than epsilon.
>From Theorem 5, for any delta>0, we can find large enough t that p(1,epsilon,t) > 1-delta. But then from the above observation, we can find a<1 which is close enough to 1 that |p(1,epsilon,t) - p(gamma,epsilon,t)| < delta whenever gamma>a.

It follows from the triangle inequality that p(gamma,epsilon,t) > 1-2*delta whenever gamma>a. In particular, for gamma>a,

P[bankrupt, or inf_t K(t) < epsilon] >= 1-2*delta.

Since delta>0 was arbitrary, the result follows. [#]

REMARK: Even if (10) holds with r=0 and mu>0, then for large enough sigma and small enough gamma, there could be a positive probability of avoiding eventual bankruptcy. However, this probability would be usually be extremely small, since (by Theorem 5) it arises solely because of the very subtle difference between max[SP(t),0] and SP(t).

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