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# OPTIMAL METROPOLIS SCALING: PREVIOUS PROOFS AND POSSIBLE GENERALIZATIONS

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# ON MCMC ALGORITHMS

- THE SAMPLING PROBLEM
- APPLICATIONS

## OUR CASE STUDY : THE RWM

- MCMC framework: RWM, MALA, HMC...
- A SIMPLE APPROACH : THE LEAPFROG DIAGRAM

“We start somewhere in the state space, jump randomly in a certain perimeter defined by a *proposal distribution*, look at the new state we obtain, and decide to move to it with a probability describing how closer this new position is to a 'high-density zone' of the *target distribution*.”

“In conclusion, we start somewhere and randomly explore the state space in directions of higher density of target distribution we wish to emulate.”

# THE LITERATURE ON OPTIMALITY

- iid *target distribution*: Roberts, Gelman, Gilks
- i -not i- d *target distribution*: Bédard, Rosenthal
- infinite-dimensional *target distribution*: Stuart, Mattingly, Pillai
- Toward a more general *target distribution*? Why? How?
- Relevance of the original paper in understanding subsequent research

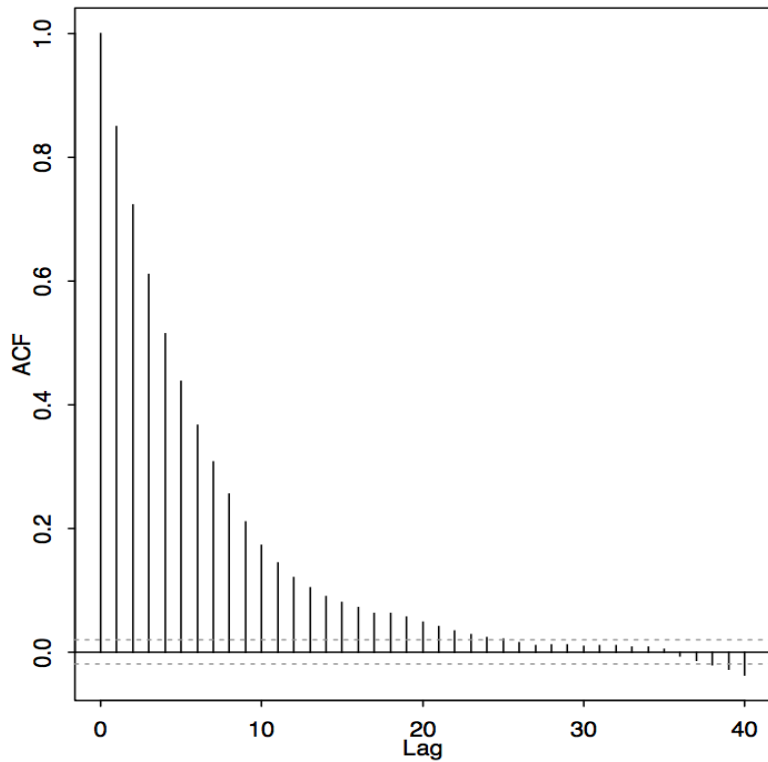
# RGG'S PROOF : THE LARGE SCHEME

- *Discrete Markov Chain* ----- Time-Space Rescaling -----> *Continuous Langevin Dynamic*
- Ethier and Kurtz: Stochastic processes , Skorokhod CV  
↔  
Generators --- LI CV
- Gross points: Taylor expansions...
- Finer points: equivalency in... the Skhorokhod topology  
... of deterministic (linear) and stochastic (Poisson-process) acceleration  
... by time-elasticity of the topology

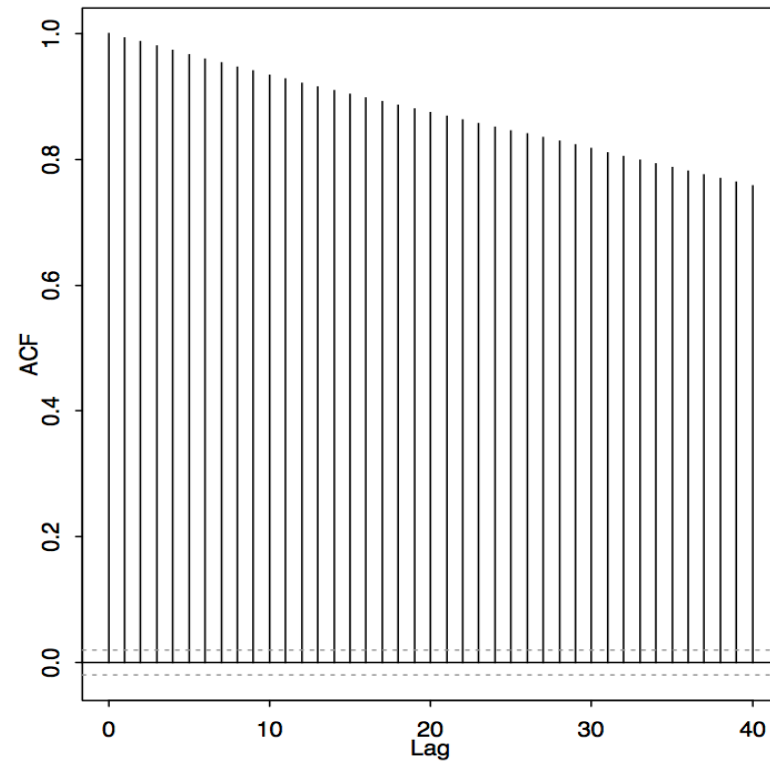
# IN PRACTICE

- TUNING : the SIGMA-JUMP for a 0.234 ACCEPTANCE RATE

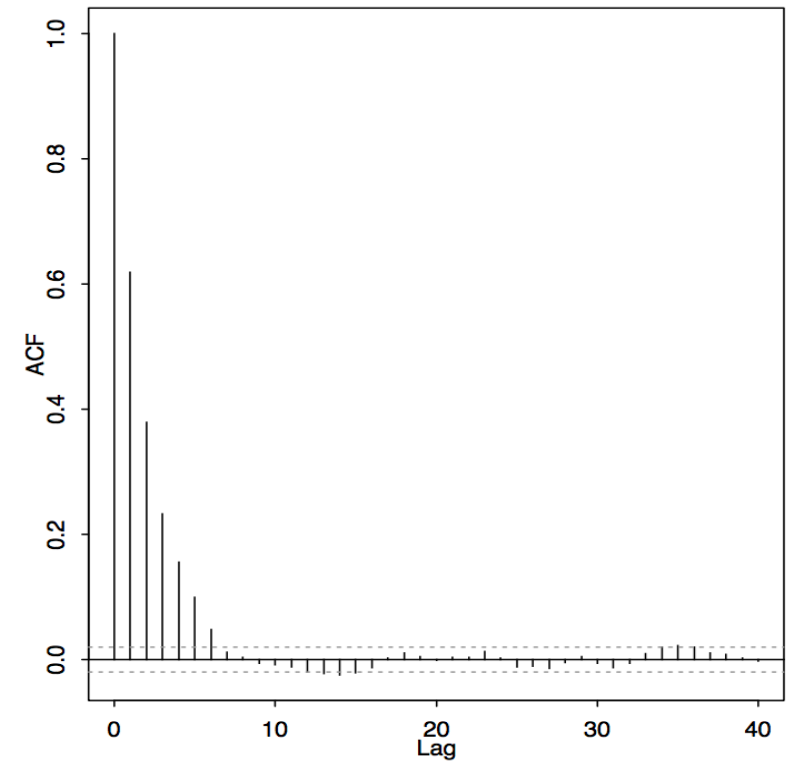
(a) Proposal variance too large



(b) Proposal variance too small



(b) Proposal variance approximately optimised



From *Roberts and Rosenthal, 2001*

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