1) Fixed Change-point K=3

Density of Height with Running Length (N) = 300
Density of Height with Running Length (N) = 1000
Density of Height with Running Length \((N) = 10000\)
Density of Height with Running Length \((N) = 15000\)
Comparsion of density of height

Running Length=300

N = 900   Bandwidth = 0.2666

Running Length=1000

N = 3000   Bandwidth = 0.2135
Running Length=10000

N = 30000   Bandwidth = 0.1387

Density

0.0 0.2 0.4 0.6

0 1 2 3 4

0 1 2 3 4

N = 30000   Bandwidth = 0.1387
Running Length=15000

N = 45000   Bandwidth = 0.1265
Result:

For $N < 10000$, the modes are not very clear. Thus, running length is too short if it is less than 10000. Besides, the modes tends to be clearer as running length reaches and beyonds 10000.
Density of Position with Running Length \( N = 300 \)

\[ \text{Running Length} = 300 \]

\[ N = 300 \quad \text{Bandwidth} = 0.4784 \]
Density of Height with Running Length (N) = 1000

Running Length = 1000

N = 1000  Bandwidth = 0.3127
Density of Height with Running Length (N) = 10000

Running Length = 10000

N = 10000  Bandwidth = 0.3358
Density of Height with Running Length \( (N) = 15000 \)

\[ \text{Running Length} = 15000 \]

\[ \text{N} = 15000 \quad \text{Bandwidth} = 0.2475 \]
ACF of S2
Result:

1) The density of $s_2$ and $s_3$ overlap around 1900, which makes it be an important changepoint. The plot shows that $h_2$ and $h_3$ also jumps to $h_1$ at same running length. Thus, the jump of position may caused by the change of height.

2) The distance between the mode of $s_1$ and $s_2$ dataset gets farther as running time increases. Besides, one mode finally "wins" with the highest probability for the choice of both $s_1$ and $s_2$. Thus, the choice of $s_1$ and $s_2$ converges to the "true" value with the increase of running length.
2) Varied K

Histogram of $K$ with Running Length = 300

<table>
<thead>
<tr>
<th>$K$</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Histogram of K with Running Length = 10000

<table>
<thead>
<tr>
<th>K</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>4</td>
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<tr>
<td>6</td>
<td></td>
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<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Histogram of K with Running Length = 15000
Result:

The range of k values tends to be wider as running length increases. Besides, the proportion of extreme values tends to be smaller as running length increases and $k = 3$, $k = 4$ are always the most frequent choice of k value.
Density of Height with $K=3$ and Running length=15000

**h1**

![Graph of density distribution] N = 3746  Bandwidth = 0.04724

**h2**

![Graph of density distribution] N = 3746  Bandwidth = 0.02359
Density of Height

N = 11238   Bandwidth = 0.163
Running Length = 15000

![Graph showing running length vs height]
Series $H3[, 2]$
Running Length=15000

Position

Running length
Result:

Compared with fixed $k = 3$ case, $s_2$ jumps close to $s_1$ more frequently as well as $h_3$, $h_2$ and $h_1$, although the density of height are similar.