Recall that we were considering the following game (to be referred to as the original game). A starts with $a$ pennies, and B starts with $8 - a$ pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then B gives one penny to A. If it comes up 3, 4, 5, or 6, then A gives one penny to B. This is repeated until either A or B wins all the pennies. That person is the “winner”. Recall that we wrote $s(a)$ for the chance that A wins this game, starting with $a$ pennies.

What follows is a “reminder” of how we derived a formula for $s(a)$.

**Step #1.** Obviously $s(0) = 0$ and $s(8) = 1$.

**Step #2.** By considering what happens on the first bet, we see that

$$s(a) = \frac{1}{3}s(a + 1) + \frac{2}{3}s(a - 1),$$

for $a = 1, 2, 3, 4, 5, 6, 7$.

**Step #3.** Since $s(a) = \frac{1}{3}s(a) + \frac{2}{3}s(a)$, this last formula can be re-written as

$$s(a + 1) - s(a) = 2(s(a) - s(a - 1)).$$

**Step #4.** Hence, setting $x = s(1)$, we see that

$$s(1) - s(0) = x, \quad s(2) - s(1) = 2x, \quad s(3) - s(2) = 4x, \quad \text{etc.}$$

and in general that

$$s(a + 1) - s(a) = 2^ax$$

for $a = 0, 1, 2, 3, 4, 5, 6, 7$.

**Step #5.** It follows that, for $a = 0, 1, 2, 3, 4, 5, 6, 7, 8$,

$$s(a) = s(a) - s(0) = (s(a) - s(a - 1)) + (s(a - 1) - s(a - 2)) + \ldots + (s(1) - s(0))$$

$$= (2^{a-1} + 2^{a-2} + \ldots + 2 + 1) x = (2^a - 1) x.$$