SCI 199Y: Random Walks and Mathematical Discovery

Math exercise, week 12.

This week we will consider betting strategies for our gambling game.

Specifically, suppose that **A** starts with 6 pennies, and **B** starts with 2 pennies. A fair 6-sided die is repeatedly rolled. Before each roll, player **A** is allowed to choose a positive integer "m" which is the number of pennies to be bet. (Note that m cannot be more than the number of pennies that **A** has left!) Then, if the die comes up 1 or 2, then **B** gives m pennies to **A**. If it comes up 3, 4, 5, or 6, then **A** gives m pennies to **B**. This is repeated until either **A** or **B** wins all the pennies. That person is the "winner".

Of course, if **A** always choses $m = 1$, then this is our "original game" from before. In that case, we know that **A** has probability $s(6) = \frac{2^6-1}{255} = \frac{63}{255} \doteq 0.247$ of winning.

Questions:

Form a group of 2 or 3 students. As a group, using pennies and dice, try to estimate the probability that **A** will win the game (starting with $a = 6$ pennies), if they adopt each of the following strategies for how much to choose to bet.

(a) Bet $m = 2$ pennies on each roll.

(b) Bet $m = 2$ pennies on the first roll. Then, if **A** loses the first roll, bet $m = 4$ pennies on the second roll. (After that the game will be over, one way or another!)

(c) Bet $m = 2$ pennies on the first roll. Then, if **A** loses the first roll, bet $m = 1$ penny on each roll after the first.

(d) Any other strategies you think might be helpful to **A**.

Which strategy is the best for **A** to use??

(If you wish, you may also try to figure out mathematical formulas for these probabilities, but you don't have to.)

What if the game is modified (more pennies and/or different probability of winning each bet)? Would that affect the choice of a best strategy for **A**?

Casino: Next week, we will hold a "casino" of sorts. I will set up various betting games involving strategy choices (such as the game described here), and will allow you to bet on them. There will be prizes for the winners!