SCI 199Y: Random Walks and Mathematical Discovery

Math exercise, week 14.

Index Card. On the index card provided, please write (a) your name; (b) the highest-level math course you took or are currently taking (e.g. "Grade 11 math" or "MAT 135Y"); (c) your self-assessed mathematical ability compared to other students in this class, on a scale of 0 (lowest) to 10 (highest), being careful not to undervalue yourself; (d) the extent to which you enjoy trying to solve math problems, on a scale of 0 (lowest) to 10 (highest); (e) your birthday (month and day only); (f) your mother's birthday (month and day only). [Note: The reason for these last two items will be made clear later!] Then hand the card back to me. Thanks!

This week we will begin a study of "simple symmetric random walk". This topic is related to the betting game from last semester, but has a different emphasis.

The idea is to continually make fair bets of one penny each, and to keep track of the long-term behaviour. But unlike last semester, you have (in theory) an unlimited number of pennies and no limit to how many pennies you can win.

Choose a partner. With your partner, try modeling this phenomenon as follows. (You will need one 6-sided die, one penny, and two fresh sheets of paper.)

(1) On a blank sheet of paper, draw a row of (at least) 21 small "boxes", labeled (from left to right) as -10, -9, -8, ..., -1, 0, 1, 2, ..., 9, 10.

(2) On a separate piece of paper, make up a chart with three columns, labeled "1", "4", and "10", with plenty of room in each column.

(3) Place a penny in the box labeled "0" (i.e., in the middle box).

(4) Repeatedly roll a six-sided die. Each time the die comes up 4, 5, or 6, move the penny one box to the right. Each time the die comes up 1, 2, or 3, move the penny one box to the left.

(5) On the separate sheet of paper from step (2), record the position of the penny after the first, fourth, and tenth rolls (i.e., write one number in each of the three columns).

(6) After the tenth roll, go back to step (3) and start over again. Repeat this experiment at least 20 times.

(7) Once you have enough data (i.e., at least 20 numbers in each of the three columns), then analyze the data as follows. For each of the three columns, compute the average of the numbers, and also the average of the squares of the numbers. Try to find a pattern if you can!

(8) For each of the last two columns, compute the fraction of entries which are "0". Use this to estimate the *probability* that simple symmetric random walk will return to 0 after 4 rolls, and after 10 rolls.

(9) If you have time, try to guess how the answers in (7) and (8) above would change if we recorded the position after n rolls (so far we've just considered $n = 1$, $n = 4$, and $n = 10$). If you're feeling ambitious, try to explain the results mathematically.

Note: Be sure to bring your 21-box sheet, and also your data from this experiment, back to class next week.

Reading Assignment: Read the five-page excerpt from "Women and Gender: A Feminist Psychology" by R. Unger and M. Crawford. Also read over the brief biographical excerpts about the female mathematics geniuses Emmy Noether and Olga Taussky-Todd.

Note: For information about other female mathematicians, Web-minded students may wish to consult the URL

http://www-groups.dcs.st-and.ac.uk/∼history/Indexes/Women.html