SCI 199Y: Random Walks and Mathematical Discovery

Group math exercise, week 3.

This week we will begin the process of solving mathematically the probabilities that came up in the betting game.

> **Comment:** This is not an easy question, and you are not expected to necessarily be able to solve it right away! That is often true in the study of mathematics (unlike high-school mathematics), and it should not be a source of frustration.

You will be divided into groups of about 4 people. Introduce yourselves. Then consider the following game:

1. Give a pennies to **A**, and $8 - a$ pennies to **B**.
2. Roll a fair 6-sided die. If it comes up 1 or 2, then **B** gives one penny to **A**. If it comes up 3, 4, 5, or 6, then **A** gives one penny to **B**.
3. Repeat step 4 until either **A** or **B** wins all the pennies. That person is the "winner".

(If $a = 6$, then this is the game that we played last time.)

Suppose we write $s(a)$ for the chance that **A** wins this game, starting with a pennies. We would like to find a formula for $s(a)$.

Question #1: (easy) What are $s(0)$ and $s(8)$?

Question #2: (tricky) Can you find some equation relating the three numbers $s(5)$, $s(6)$, and $s(7)$? (**Hint:** Imagine starting with $a = 6$ pennies for **A**, and think about the possibilities for what happens on the first bet.)

If you have time, then carry on:

Question #3: (no harder) More generally, can you find an equation relating the three numbers $s(a-1)$, $s(a)$, and $s(a+1)$, for any number a between 1 and 7?

Question #4: (hard!) Putting all of this together, can you find a formula for $s(a)$?

While you are working on all of this, you may also wish to consider the following (somewhat philosophical) question:

Question #5: Is it possible that the game could go on forever, with neither person ever winning?