

SCI 199Y: Random Walks and Mathematical Discovery

Group math exercise, week 6.

This week we will again continue the process of solving mathematically the probabilities that came up in the betting game. Return to your same groups from the math exercise from last time. Re-introduce yourselves.

Group Processing: Before considering the math questions below, spend a few minutes on “group processing”. Discuss the following questions: How well are you working as a group? Are you making sure that all group members understand each idea? Do you feel comfortable asking questions when you do not understand? When you get stuck as a group, do you focus on clarifying what you know, what you want to know, and possible strategies to make further progress? Try to list ways in which your group is working together well, and ways in which you would like to improve.

Once you have properly finished your group processing, then consider the math questions below.

Recall that we were considering the following game. **A** starts with a pennies, and **B** starts with $8 - a$ pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then **B** gives one penny to **A**. If it comes up 3, 4, 5, or 6, then **A** gives one penny to **B**. This is repeated until either **A** or **B** wins all the pennies. That person is the “winner”. Recall that we wrote $s(a)$ for the chance that **A** wins this game, starting with a pennies.

Recall from last time that we now know that $s(0) = 0$ and $s(8) = 1$. We also know that $s(a) = \frac{1}{3}s(a+1) + \frac{2}{3}s(a-1)$, for each integer a between 1 and 7. We also know that this equation is equivalent to saying that

$$s(a+1) - s(a) = 2(s(a) - s(a-1)) ,$$

for each integer a between 1 and 7.

Working cooperatively with your group members, consider the following questions.

Question #1: Let $x = s(1)$, an unknown quantity. In terms of x , what is $s(1) - s(0)$? What is $s(2) - s(1)$? What is $s(3) - s(2)$? What is $s(8) - s(7)$?

Question #2: In terms of x , what is $s(2)$? What is $s(3)$? What is $s(4)$? What is $s(8)$?

Question #3: Since we know $s(8) = 1$, therefore what does x have to be?

Question #4: Putting all of this together, can you find a formula for $s(a)$?