

SCI 199Y: Random Walks and Mathematical Discovery

Math exercise, week 7.

This week we will try writing a complete description of the mathematics for the betting game that we have been considering.

Recall that we were considering the following game. **A** starts with a pennies, and **B** starts with $8 - a$ pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then **B** gives one penny to **A**. If it comes up 3, 4, 5, or 6, then **A** gives one penny to **B**. This is repeated until either **A** or **B** wins all the pennies. That person is the “winner”. Recall that we wrote $s(a)$ for the chance that **A** wins this game, starting with a pennies.

Recall from last time that we (finally!) figured out the formula

$$s(a) = \frac{2^a - 1}{255}$$

Working either alone, or cooperatively with a group of your own choosing (of any size, provided that everyone participates together!), perform the following task.

The Task: Write a short note (probably between 1 and 3 pages) explaining how we derived this formula for $s(a)$. Your note should be appropriate for inclusion in an elementary textbook on probability theory, or as a report for a company interested in this formula. It should explain the mathematics of how to find the solution, providing as much detail and as much understanding as possible. It may also consider related matters (e.g. why did we need to use such an indirect method?) as you feel is appropriate.

Note: The entries will be judged by a jury consisting of several class members who have identified themselves as being weak mathematically. There will be prizes for the winners. Thus, your note should explain things clearly enough that the jury is impressed!

Note to Jury Members: While the other students are writing their notes, you should be discussing what aspects of the mathematics for the formula for $s(a)$ you found confusing, what things you hope to see explained in the notes, and what criteria you will use to judge the notes.