SCI 199Y: Random Walks and Mathematical Discovery

Math exercise, weeks 8 and 9.

Recall that we were considering the following game (to be referred to as the original game). **A** starts with a pennies, and **B** starts with $8 - a$ pennies. A fair 6-sided die is repeatedly rolled. If it comes up 1 or 2, then **B** gives one penny to **A**. If it comes up 3, 4, 5, or 6, then **A** gives one penny to **B**. This is repeated until either **A** or **B** wins all the pennies. That person is the "winner". Recall that we wrote $s(a)$ for the chance that **A** wins this game, starting with a pennies.

Recall that we have derived the formula

$$s(a) \;=\; \frac{2^a - 1}{255}$$

You will be assigned into a math-ability-heterogeneous group of about 4 or 5 students. Working cooperatively with your group, consider the following questions related to extending this formula.

Question #1: Suppose that there are a total of "N" pennies (instead of "8"), so that **A** starts with a pennies and **B** starts with $N - a$ pennies, for some integer a between 0 and N. All other rules remain the same. What is the formula for $s(a)$ for this modified game?

Question #2: Suppose that on each bet, player **A** has probability "p" (instead of "1/3") of winning one penny (and probability $1-p$ of losing one penny), for some number p between 0 and 1. All other rules remain the same. What is the formula for $s(a)$ for this modified game?

Question #3: In the original game, suppose we write $r(b)$ for the chance that player **B** wins the game, if **B** starts with b pennies and **A** starts with $8 - b$ pennies. What is a formula for $r(b)$? [Hint: the answer to Question #2 may help here.]

Question #4: In the original game, is it possible that the game will go on forever, with no one ever winning? [Note: This question can be considered in several different ways. The answer to Question #3 may help.]

Question #5: What if player **A** starts with a pennies, but player **B** has an infinite amount of money. What is the probability that the game will go on forever, with **A** never running out of money?