

SCI 199Y: Random Walks and Mathematical Discovery

Discussion questions, week 8.

You have read the one-page excerpt about prime numbers from “What is Mathematics?”, by R. Courant and H. Robbins. You have also read the four-page excerpt from “The psychology of invention in the mathematical field”, by J. Hadamard.

You will be assigned into a math-ability-heterogeneous group of about 4 or 5 students. Working cooperatively with your group, consider the following questions.

Question #1. Clarify among yourselves the meaning of “prime number”, and what it means to say “There are infinitely many primes”. Did you already know that there were infinitely many primes? Do you find it surprising?

Question #2. Explain in detail Euclid’s proof that there are infinitely many primes. Make sure everyone in the group understands it well.

Question #3. Euclid discovered this proof during the third century B.C., yet it is still well understood (and loved!) by mathematicians today. Why is this remarkable?

Question #4: A pair of numbers are called “twin primes” if they differ by 2 and are both prime. For example, the three pairs (3, 5), (11, 13), and (29, 31) are all twin primes. Do you think there are infinitely many twin primes? Just give your best guess, and then move on. [We’ll discuss the answer to this question later; it is likely to surprise you!]

Question #5: Discuss at some length the points made by Hadamard (a very famous French mathematician, writing in 1945) about the way he thinks while doing mathematical research. What overall conclusions can be made? What are the implications for the teaching of mathematics?

Assignment for next week: Read the excerpt from “Surely you’re joking, Mr. Feynman”, by Richard P. Feynman (a Nobel Prize-winning theoretical physicist), and also the excerpt from the interview with David Blackwell in “Mathematical People” (D.J. Albers, ed.). What attitudes do these two mathematical geniuses express towards mathematical research? Do any of these attitudes surprise you? Do they have any implications for mathematics education?