Name: ______________________________ Student #: ________________________

Program & Year: ____

STA 447/2006S, Spring 2001: Test #1

(Thursday, February 15, 2001. Time: 60 minutes.)

(Questions: 4; Pages: 4; Total points: 45.)

NO AIDS ALLOWED.

1. (10 points) Consider a (discrete-time) Markov chain on the state space $S = \{1, 2, 3\}$ such that

$$P = \begin{pmatrix} 1/5 & 2/5 & 2/5 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$

Suppose that $\mu_1^{(0)} = 1$, and $\mu_2^{(0)} = \mu_3^{(0)} = 0$. Compute $\mu_1^{(2)}$. (Explain your reasoning.)

2. (10 points) Give (with explanation) an example of a valid transition matrix P for a (discrete-time) Markov chain on the state space $S = \{1, 2, 3\}$, with the property that

$$0 < f_{12} < 1\,.$$

3. (10 points) Let $\{N(t)\}_{t\geq 0}$ be a Poisson process with constant intensity $\lambda > 0$. Compute (with explanation) the conditional probability

$$\mathbf{P}\big(N(2.6) = 2 \;\big|\; N(2.9) = 2\big)\,.$$

4. (15 points) Consider a (discrete-time) Markov chain on the state space $S = \{1, 2, 3, 4\}$ such that

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

Prove or disprove the following assertion:

$$\lim_{n \to \infty} p_{ij}(n) = 1/4 \,, \qquad \text{for all } i, j \in S \,.$$