1. (10 points) Consider a (discrete-time) Markov chain on the state space \( S = \{1, 2, 3\} \) such that
\[
P = \begin{pmatrix}
1/5 & 2/5 & 2/5 \\
1/3 & 0 & 2/3 \\
1/2 & 1/2 & 0
\end{pmatrix}.
\]
Suppose that \( \mu_1^{(0)} = 1 \), and \( \mu_2^{(0)} = \mu_3^{(0)} = 0 \). Compute \( \mu_1^{(2)} \). (Explain your reasoning.)
2. (10 points) Give (with explanation) an example of a valid transition matrix $P$ for a (discrete-time) Markov chain on the state space $S = \{1, 2, 3\}$, with the property that

$$0 < f_{12} < 1.$$
3. (10 points) Let \( \{N(t)\}_{t \geq 0} \) be a Poisson process with constant intensity \( \lambda > 0 \). Compute (with explanation) the conditional probability

\[
P(N(2.6) = 2 \mid N(2.9) = 2).
\]
4. (15 points) Consider a (discrete-time) Markov chain on the state space $S = \{1, 2, 3, 4\}$ such that

$$P = \begin{pmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} & 0 \\
0 & \frac{1}{2} & 0 & \frac{1}{2} \\
0 & 0 & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}.$$ 

Prove or disprove the following assertion:

$$\lim_{n \to \infty} p_{ij}(n) = \frac{1}{4}, \quad \text{for all } i, j \in S.$$