1. (10 points) Consider a single-server queue with interarrival time distribution $\text{Exp}(\lambda)$, and service time distribution $\text{Unif}[0,10]$. Let $W_n$ be the waiting time of the $n^{th}$ customer. Give (with explanation) necessary and sufficient conditions on $\lambda$ such that $W_n \to \infty$ in probability.
2. (10 points) Let $\{N(t)\}$ be a non-arithmetic renewal process with finite mean inter-arrival time $\mu$. Fix $h > 0$. Compute (with explanation) the limit

$$\lim_{t \to \infty} \left( \frac{N(t+h) - N(t)}{t} \right)^2.$$
3. (15 points) Let $a$ and $c$ be positive integers, with $0 < a < c - 1$. Consider the Gambler’s Ruin Markov chain $\{X_n\}$ on $\{0, 1, \ldots, c\}$ with $p = \frac{1}{2}$, so that $X_0 = a$, and $p_{i,i+1} = p_{i,i-1} = \frac{1}{2}$ for $1 \leq i \leq c - 1$ and $p_{00} = p_{cc} = 1$. Define the stopping time $U$ by $U = \min\{n \geq 1; X_n = a + 1\}$.

(a) Show that $\{X_n\}$ is a martingale.

(b) Prove or disprove that $\mathbf{E}[X_U] = \mathbf{E}[X_0]$.

(c) Can the Optional Stopping Theorem (or its Corollary) be applied to this process $\{X_n\}$ and stopping time $U$? (Explain your answer.)
4. (15 points) Consider simple symmetric random walk \( \{X_n\} \) on the set of all integers \( \mathbb{Z} \), with \( X_0 = 0 \). Let \( T_2 = \min\{n \geq 1; X_n = 2\} \). Prove or disprove that

\[
\lim_{M \to \infty} \mathbb{E}[X_M | T_2 > M] = -\infty.
\]

[Hint: You may wish to set \( S = \min(T_2, M) \) and use the Law of Total Probability.]