Name: ______________________________ Student #: ________________________

STA 447/2006S, Spring 2001: Test #2

(Thursday, March 29, 2001. Time: 60 minutes.)

(Questions: 4; Pages: 4; Total points: 50.)

NO AIDS ALLOWED.

1. (10 points) Consider a single-server queue with interarrival time distribution $\mathbf{Exp}(\lambda)$, and service time distribution $\mathbf{Unif}[0, 10]$. Let W_n be the waiting time of the n^{th} customer. Give (with explanation) necessary and sufficient conditions on λ such that $W_n \to \infty$ in probability.

2. (10 points) Let $\{N(t)\}$ be a non-arithmetic renewal process with finite mean interarrival time μ. Fix $h > 0$. Compute (with explanation) the limit

$$\lim_{t\to\infty} \left(\frac{N(t+h) - N(t)}{t} \right)^2 .$$

3. (15 points) Let a and c be positive integers, with $0 < a < c - 1$. Consider the Gambler's Ruin Markov chain $\{X_n\}$ on $\{0, 1, \ldots, c\}$ with $p = \frac{1}{2}$, so that $X_0 = a$, and $p_{i,i+1} = p_{i,i-1} = \frac{1}{2}$ for $1 \leq i \leq c - 1$ and $p_{00} = p_{cc} = 1$. Define the stopping time U by $U = \min\{n \geq 1\,;\ X_n = a + 1\}$.

(a) Show that $\{X_n\}$ is a martingale.

(b) Prove or disprove that $\mathbf{E}[X_U] = \mathbf{E}[X_0]$.

(c) Can the Optional Stopping Theorem (or its Corollary) be applied to this process $\{X_n\}$ and stopping time U? (Explain your answer.)

4. (15 points) Consider simple symmetric random walk $\{X_n\}$ on the set of all integers $\mathbf{Z}$, with $X_0 = 0$. Let $T_2 = \min\{n \geq 1\,;\; X_n = 2\}$. Prove or disprove that

$$\lim_{M \to \infty} \mathbf{E}[X_M \,|\, T_2 > M] \;=\; -\infty\,.$$

[Hint: You may wish to set $S = \min(T_2, M)$ and use the Law of Total Probability.]