Name: ______________________________ Student #: ________________________

STA 447/2006S, Spring 2002: Test #2

(Thursday, March 28, 2002. Time: 80 minutes.)

(Questions: 5; Pages: 3; Total points: 50.)

NO AIDS ALLOWED. You may use results from class.

1. (10 points) Let $s(p, c, a)$ be the gambler's ruin probability, i.e. the probability that simple random walk with parameter p, started at $X_0 = a$, will hit c before it hits 0. Compute (with explanation) the limit $\lim_{n\to\infty} s(p, 2n, n)$, for $0 < p < 1$.

2. (10 points) Let $\{\hat{X}_n\}_{n=0}^{\infty}$ be a discrete-time Markov chain on the state space $S = \{1, 2, 3\}$, with $\hat{X}_0 = 1$, and with transition probabilities given by

$$(\, p_{ij}\,) \;=\; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/6 & 1/2 \end{pmatrix}.$$

Let $\{X(t)\}_{t\geq 0}$ be the Exponential(λ) holding time modification of $\{\hat{X}_n\}$. Let $T_3 = \min\{t \geq 0 : \; X(t) = 3\}$. Compute (with explanation) the expected value of T_3.

3. (10 points) Let $\{N(t)\}_{t \geq 0}$ be a Poisson process with parameter $\lambda = 3$. Let $X = N(8) - N(5)$, and let $Y = N(7) - N(2)$. Compute (with explanation) the value of $E[XY]$.

4. (10 points) Let $\{X(t)\}_{t \geq 0}$ be a continuous-time Markov process on the state space $S = \{1, 2, 3, 4, 5\}$. Suppose it is known that for $0 \leq t \leq 0.03$,

$$P(X(t) = 2 \mid X(0) = 1) = 5t + 4t^2 + e^{3t} - 1\,.$$

Let $G = (\, g_{ij} \,)$ be the generator for this process. Compute g_{12}.

5. (10 points) Let $\{Y_n\}_{n=0}^{\infty}$ be i.i.d. $\sim$ **Uniform**$[0,10]$. Let $T_0 = 0$, and let $T_n = Y_1 + Y_2 + \ldots + Y_n$ for $n \geq 1$. Let $p = P[\,\exists n \geq 1 : \; 1234.5 < T_n < 1234.6]$. Find (with explanation) a good approximation to p.