Errata for the SECOND EDITION of “A First Look at Rigorous Probability”,

(Note: throughout, “line \( -x \)” means \( x \) lines from the bottom.)

Errata to Seventh Printing, 2016 (to be corrected in Eighth Printing, 2020):

[With thanks to Damir Ferizovic, Daehyun Kim, and Martin Yang.]

- Page 33, line 7: after “for fixed Borel \( S \subseteq \mathbb{R} \)” add “with \( P(Y \in S) > 0 \)”. 
- Page 36 middle: replace the second \( \sum_{k=1}^{\infty} P(B_{2^k}) \) with \( \sum_{k=1}^{\infty} (1/4) \).
- Page 40, Exercise 3.6.18, line 2: better to change “lim” to “\( \lim \inf \)” to avoid issues of the existence of the limit. (Alternatively, the current definition of \( S_x \) can be taken to mean the limit exists and is \( \leq x \).)
- Page 53, Exercise 4.5.10, first line: replace “i.i.d.” with “independent”, to make the statement stronger, and also because i.i.d. hasn’t been introduced yet.
- Page 76, equation (7.2.2): should end with a comma, not a period.
- Page 77, lines 11 and 12 (twice): replace the word “obtaining” with the word “reaching” (for greater clarity).
- Page 85, line \(-6\): strictly speaking, \( \{I_{ij}^{(1)}\}_{i,j \in S} \) should be \( \{I_{ij}^{(1)}\}_{j \in S} \).

Errata to Sixth Printing, 2013 (to be corrected in Seventh Printing, 2016):

[With thanks to Anthony Brooms, Geoffrey Brown, Julian Ziegler Hunts, Owen Lyne, Byron Schmuland, Adrian Torchiana, and Peter Tye.]

- Page 3 middle, change “not used elsewhere in this book” to “not required elsewhere in this book”.
- Page 5 middle, change “mathematical rigorous” to “mathematically rigorous”.
- Page 21, line 8: “\( A \subseteq \mathcal{F} \)” should be “\( A \in \mathcal{F} \)”.
- Page 23, line \(-3\): “a algebra” should be “an algebra”.
- Page 29, eqn (3.1.2): “\( X(w) \)” should be “\( X(\omega) \)”.
- Page 40, Exercise 3.6.13, line 2: “\( P \)” should be “\( \mathbb{P} \)”, and “\( X \)” should be “\( X_n \)”, and “for all \( i \)” should be “for all \( n \)”.

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• Page 46: omit the last two sentences (since we are already assuming that the \( X_n \) are non-negative).

• Page 57, line -4: “\( \mu_y \)” should be “\( \mu_Y \)”.

• Page 87, second half: “Sterling” should be “Stirling” (twice).

• Page 71, Exercise 6.3.1: Replace the first sentence by, “Let \( \mu \) have density \( 4x^31_{0<x<1} \), and let \( \nu \) have density \( \frac{1}{2}x1_{0<x<2} \).”

• Page 119, lines 1–2: since \( Y \) is actually defined as a function on \((0,1)\), it is better to replace “\( m(Y(n+1) - Y(n)) < \infty \) discontinuities of size \( \geq 1/m \) within the interval \((n,n+1)\)” with “\( m[Y(1 - \frac{1}{n}) - Y(\frac{1}{n})] < \infty \) discontinuities of size \( \geq 1/m \) within the interval \((\frac{1}{n}, 1 - \frac{1}{n})\).”

• Page 121, Exercise 10.3.2, line 2: omit the comma just before the period.

• Page 134: the last three lines of the proof of Theorem 11.2.2 are poorly phrased, since \( \phi_n(t) \) might be complex, and should be replaced by e.g.: “\( n \to \infty \). This means that for any \( \epsilon > 0 \), for sufficiently large \( n \) we have \( |nq_n| \leq \epsilon \). Hence, as \( n \to \infty \) the limit is multiplied by \( e^{nq_n} \), which becomes arbitrarily close to 1 and hence can be ignored.”

• Page 147, lines 17–18: “\( \mu \)” and “\( \nu \)” should be interchanged (twice).

• Page 178, 2/3 of the way down the page: “only if” and “if” should be interchanged.

• Page 181, Exercise 15.2.7: change “a Markov chain” to “an irreducible Markov chain”.

• Page 183, Exercise 15.3.3, line 3: change “\( A_0 \)” to “\( A_1 \)”.

• Page 187, line -12: \( \frac{1}{n} \mathbb{E} \left( Y^{(n)}_{\lceil sn \rceil} Y^{(n)}_{\lceil tn \rceil} \right) = \mathbb{E} \left( (Z_1 + \ldots + Z_{\lfloor sn \rfloor})(Z_1 + \ldots + Z_{\lfloor tn \rfloor}) \right) \)” should be \( \mathbb{E} \left( Y^{(n)}_{\lfloor sn \rfloor} Y^{(n)}_{\lfloor tn \rfloor} \right) = \frac{1}{n} \mathbb{E} \left( (Z_1 + \ldots + Z_{\lfloor sn \rfloor})(Z_1 + \ldots + Z_{\lfloor tn \rfloor}) \right) \).”

• Page 192, Exercise 15.6.8, line 2: change “\( \int_0^t bB_s \, ds \)” to “\( \int_0^t dB_s \).”

**Errata to Fifth Printing, 2011 (corrected in Sixth Printing, 2013):**

[After teaching from this book again after a five-year break, and also with thanks to Daniel Firka, Julian Ziegler Hunts, Jianlin Zou, Luis Mendo Tomás, and Sebastiaan Janssens.]

• Page 19, Exercise 2.5.6: change “finite unions” to “finite disjoint unions”, and note that \( \mathbb{P} \) is extended to \( A_n \) by finite additivity.

• Page 20: for clarity, replace “distributions” by “measures” (twice).

• Page 22, line 2: change “finite unions” to “finite disjoint unions”.

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• Page 25, line 1: “setset” should be “subset”.
• Page 35, line 4: “considering” should be “consider”.
• Page 36, line 9: missing “}”.
• Page 36, several places: omit extra \{\ldots, \}, e.g. \{H_n \cap H_{n+1}\} should be simply “H_n \cap H_{n+1}”, etc.
• Page 39, Exercise 3.6.5: for clarity, assume \mathcal{F} = 2^\Omega.
• Page 39, line 4: “P” should be \boldsymbol{P}.
• Page 40, Exercise 3.6.11: the notation “\sim” has not yet been introduced, so “X_n \sim \text{Uniform}(\{1,2,\ldots,n\})” should be replaced by \mathbf{P}(X_n = i) = 1/n for i = 1,2,\ldots,n”.
• Page 40, Exercise 3.6.13: should be moved to LATER (since it uses expectation), e.g. as Exercise 4.5.16. (And “E” should be \boldsymbol{E}, twice.) It could be replaced by e.g.

Let X_1, X_2,\ldots be defined jointly on some probability space (\Omega, \mathcal{F}, P), with \sum_{i=1}^\infty i^2 \mathbf{P}(i \leq X < i + 1) \leq C < \infty for all i. Prove that \mathbf{P}[X_n \geq n \text{ i.o.}] = 0.

• Page 46, statement of Theorem 4.2.2: assume the X_n are non-negative, and then omit “\mathbf{E}(X_1) > -\infty” (since we haven’t yet defined expected values of general random variables).
• Page 46, first line of proof: for greater clarity, replace “(3.1.6)” by “Proposition 3.1.5.(iii)”. (Similarly page 107, line 5.)
• Page 46, lines -3 and -2: “E” should be boldface (twice).
• Page 49, exercise 4.3.3(a): change “Z^+ and Z^-” to “Z^+ - Z^-”.
• Page 54, Exercise 4.5.13(d): replace “\mathbf{E}(X) < \infty” by “0 < \mathbf{E}(X) < \infty”.
• Page 58, Lemma 5.2.1: note that the converse also holds.
• Page 60: introduce the abbreviations “WLLN” and “SLLN”.
• Page 65, Exercise 5.5.9, Hint: specify that y > 0 for the first part, too.
• Page 66, Exercise 5.5.13, Hint: “r different sums” should be “r + 1 different sums”.
• Page 71, Exercise 6.3.1 is a repeat of Exercise 4.5.1 (page 52), and should be replaced by e.g.:

Let \mu have density \textstyle x^3 \mathbf{1}_{0 < x < 1}, and let \nu have density \textstyle x \mathbf{1}_{0 < x < 2}.

\begin{itemize}
  \item[(a)] Compute \mathbf{E}(X) where \mathcal{L}(X) = \textstyle \frac{1}{3} \mu + \frac{2}{3} \nu.
  \item[(b)] Compute \mathbf{E}(Y^2) where \mathcal{L}(Y) = \textstyle \frac{5}{6} \mu + \frac{1}{3} \delta_2 + \frac{1}{2} \delta_5.
  \item[(c)] Compute \mathbf{E}(Z^3) where \mathcal{L}(Z) = \textstyle \frac{3}{8} \mu + \frac{1}{8} \nu + \frac{1}{4} \delta_3 + \frac{1}{2} \delta_4.
\end{itemize}

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- Page 76, Exercise 7.2.5(d): for clarity, prepend “Use the fact that $s(c) = 1$ to”.
- Page 85, statement of Theorem 8.1.1, displayed eqn: “$nu_{i_0}$” should be “$\nu_{i_0}$”.
- Page 114, line −1: “$\beta^\alpha$” should be “$\beta^{-\alpha}$”.
- Page 115, line 1: “$t^{x-1}$” should be “$t^{\alpha-1}$”.
- Page 117, line 2: for clarity, change “distributions” to “measures”.
- Page 125, middle of page, displayed equations: final inequality ($\leq$) is actually an equality ($=$).
- Page 144, line −5: omit the second “then”.
- Page 159, Exercise 13.4.5: Assume $P\{1\} = P\{2\} = P\{3\} = 1/3$.
- Page 159, Exercise 13.4.6: Change “$E(X | G)$ and $E(Y | G)$ need not be independent” to “we might have $E(XY | G) \neq E(X | G) E(Y | G)$”.
- Page 165, line 3: “Theorem 14.1.3” should be “Theorem 14.1.5”.
- Page 171: for clarity, replace “for a submartingale, the same inequality holds” by “for submartingales, similarly inequalities hold”.
- Page 172, line −2: “Then follows” should be “It then follows”.
- Page 173, Exercise 14.4.5: Assume $E|Z| < \infty$.
- Page 181, Theorem 15.2.3: “$\rightarrow 0$” should be “$= 0$”.
- Page 187: $O(1/n)$ should be $O(1/\sqrt{n})$ (three times).
- Page 192: Exercise 15.6.7: For greater generality, replace “$X_t = at + bB_t$” by “$X_t = X_0 + at + bB_t$” in part (a), and “$N(at, b^2 t)$” by “$N(X_0 + at, b^2 t)$” in part (b).
- Page 192: Exercise 15.6.8: Replace “$\int_0^t s \, ds$” by “$\int_0^t a \, ds$”.
- Page 206 middle, “it suffice to assume” should be “it suffices to assume”.

Errata to Fourth Printing, 2010 (corrected in Fifth Printing, 2011):

[With thanks to David Alexander, Martin Hazelton, Andrea Lecchini-Visintini, Gareth Roberts, Igal Sason, Mohsen Soltanifar, and Albert Zevelev.]

- p. 13, line 3 of proof of Lemma 2.3.11: “$\mathcal{B}_m^C$” should be “$\mathcal{B}_m^C$”.
- p. 23, Exercise 2.7.3, part (a): “an semialgebra” should be “a semialgebra”.
- p. 31, line 5: “$f^{-1}((\infty, x])$” should be “$f^{-1}((\infty, x])$”.

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\[ \sum_j y_j 1_{B_j} \] should be \[ \sum_j y_j 1_{B_j} \].

- p. 48, line -5: “non non-negative” should be “not non-negative”.

- p. 66, line -8: “Holder” should be “Jensen”.

- p. 69, first line of Proposition 6.2.1: “\( \mu_I \)” should be “\( \mu_i \)”.

- p. 74, line 7: “\( F(z) \) for each \( z \in \mathbb{R} \)” should be “\( F(x) \) for each \( x \in \mathbb{R} \)”.

- p. 85, displayed equation in Theorem 8.1.1: “\( X = i_1 \)” should be “\( X_1 = i_1 \)”.

- p. 106, line 15: “integral” should be “integrable”.

- p. 106, line -1: “\( E(Y) \)” should be “\( E(Y) \)”.

- p. 136, Exercise 11.3.2: “\( Y_n \)” should be “\( Y_k \)” (twice).

- p. 137, conclusion of Theorem 11.4.1: “\( \mu \Rightarrow \mu \)” should be “\( \mu_n \Rightarrow \mu \)”.

- p. 146, line -6: “\( \mu_{sing} \)” should be “\( \mu_{sing} \)”.

- p. 155, middle: \[ \left\{ \{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbb{R}\right\} \] should be \[ \sigma\left(\left\{ \{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbb{R}\right\}\right) \].

- p. 171, line 11: “\( \mathbb{E} \left( \lim_{M \to \infty} U_M^{(a,\beta)} = \infty \right) = \infty \)” should be “\( \mathbb{E} \left( \lim_{M \to \infty} U_M^{(a,\beta)} \right) = \infty \)”.

- p. 178, line -6: \[ \left\{ (X_{t_1}, \ldots, X_{t_k}) \in H \right\} \] should be \[ \left\{ (X_{t_1}, \ldots, X_{t_k}) \in H \right\} \].

- p. 192, Exercise 15.6.8: replace “\( Z_t = \exp[-(a+\frac{1}{2}b^2)t+X_t] \)” with “\( Z_t = \exp[-2aX_t/b^2] \)”.

- p. 202, lines 14–15: to avoid subtleties about equivalent repeating decimals, perhaps replace “\( c_i = 4 \) if \( d_i = 5 \), while \( c_i = 5 \) if \( d_i \neq 5 \)” with “\( c_i = 2 \) if \( d_i \geq 5 \), while \( c_i = 7 \) if \( d_i < 5 \)”.

- p. 204, Exercise A.3.8: change “\( \sum_{i=1}^\infty \)” to “\( \sum_{i=2}^\infty \)” (twice), and “\( \int_1^\infty \)” to “\( \int_2^\infty \)”.

- p. 204, Exercise A.3.9: change “\( \sum_{i=1}^\infty \)” to “\( \sum_{i=3}^\infty \)” (twice).

- p. 207, second line of Exercise A.5.1: “equivalence class” should be “equivalence relation”.

Errata to Second Printing, 2007 (corrected in Fourth Printing, 2010):

[With thanks to Orn Arnaldsson, Bent Jørgensen, Chris Mansley, Kohei Nagamachi, Patrick Rabau, Mohsen Soltanifar, Hermann Thorisson.]
• p. 19, Exercise 2.5.6, and also p. 22, proof of Lemma 2.6.2: replace “$A_1, A_2, \ldots \in J$” by “$A_1, A_2, \ldots$ are finite unions of elements of $J$”.

• p. 23, Exercise 2.6.4: “$\mathbf{P}(\emptyset) = 1$” should be “$\mathbf{P}(\Omega) = 1$”.

• p. 23, Exercise 2.7.3, part (b): interchange “semialgebra” and “algebra”. (Also, for stylistic improvement, swap parts (a) and (b).)

• p. 30, last line of proof of Proposition 3.1.5: “$\{X \leq x\}$” should be “$\{Z \leq x\}$”.

• p. 33, line 7: second “$\mathbf{P}(X \in T)$” should be “$\mathbf{P}(Y \in S)$”.

• p. 39, Exercise 3.6.8: exercise is correct, but special cases like “$d \leq b + c - a$” and “$d > b + c - a$” should be modified.

• p. 40, Exercise 3.6.14: insert “independent” before “non-negative”.

• p. 74, the proof of Lemma 7.1.2 is sloppily written and should be replaced by:

Since $F$ is right-continuous, we have that $\inf\{x; F(x) \geq u\} = \min\{x; F(x) \geq u\}$, i.e. the infimum is actually obtained. It follows that $\phi(u) \leq x$ if and only if $u \leq F(x)$. Hence, since $0 \leq F(x) \leq 1$, we obtain that

$$\mathbf{P}(\phi(U) \leq x) = \mathbf{P}(U \leq F(x)) = F(x).$$

• pp. 113–114: “Example 9.5.9” should be “Exercise 9.5.9”, and similarly for 9.5.11 and 9.5.12.

• p. 114, Exercise 9.5.12: “characteristic function” should be “moment generating function”.

• p. 118 middle, the three lines following Figure 10.1.2: replace “$g$” by “$f$” (five times).

• p. 118, line –5: “$F(w) \geq b$” should be “$F(z) \geq b$”.

• p. 121, Exercise 10.3.5: “four conditions” should be “five conditions”.

• p. 126, statement of Lemma 11.1.2: “$\phi(t)$” should be “$e^{itx}$”.

• p. 128, line –7: “$\pi$” should be removed from the equation.

• p. 131, line 9: “$\lim$” should be “$\lim_k$” (twice); “$F_n$” should be “$F_{n_k}$” (twice); and “$\mu_n$” should be “$\mu_{n_k}$”.

• p. 136, lines 3–4: the sentence in brackets is somewhat misleading and should be revised or omitted.

• p. 136, eqn (11.3.1): “$1_{|Z_{n_k} \geq \epsilon s_n}$” should be “$1_{|Z_{n_k} \geq \epsilon s_n}$”.

• p. 166, lines 6–7: “bets $1$ on tails, then if they win they bet $2$ on heads” should be “bets $1$ on heads, then if they win they bet $2$ on tails”.

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• p. 173, Exercise 14.4.1: should say \( \mathbf{P}(Z_i = 1) = \mathbf{P}(Z_i = 0) = 1/2 \), and \( X_1 = 2Z_1 - 1 \).

• p. 174, Exercise 14.4.12(a), Hint: \( \mathbf{P}(\tau \geq 3m) \) should be \( \mathbf{P}(\tau > 3m) \).


[With thanks to Joe Blitzstein, Saad Siddiqui, Emil Zeuthen.]

• p. 9, line –10: “all intervals” should be “all subsets”.

• p. 18, eqn. (2.5.2): “\( \mathbf{P}(B) \)” should be “\( \mathbf{P}(B) \)”.

• p. 19, the last sentence in the proof of corollary 2.5.4 is questionable (since we may have \( D_n \not\in \mathcal{J} \)), and should be replaced by:

It then follows from (2.5.5) that

\[
\mathbf{P}\left( \bigcup_n B_n \right) = \mathbf{P}\left( \bigcup_n D_n \right) = \mathbf{P}\left( \bigcup_{i=1}^{k_n} J_{ni} \right) = \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}).
\]

On the other hand,

\[
B_n = \bigcup_{m \leq n} \bigcup_{i=1}^{k_m} (J_{mi} \cap B_n)
\]

and the union is disjoint, with \( J_{ni} \subseteq B_n \), so

\[
\mathbf{P}(B_n) = \sum_{m \leq n} \sum_{i=1}^{k_m} \mathbf{P}(J_{mi} \cap B_n) \geq \sum_{i=1}^{k_n} \mathbf{P}(J_{ni} \cap B_n) = \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}),
\]

and hence

\[
\sum_n \mathbf{P}(B_n) \geq \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}) = \mathbf{P}\left( \bigcup_n B_n \right).
\]

• p. 20, eqn. (2.5.10): \( (\infty, x] \) should be \( (-\infty, x] \).

• p. 22, line –10: \( P_1 \)” should be “\( P_1 \)” , and \( P_2 \)” should be “\( P_2 \)”.

• p. 151, first line of Section 13.1: “We being” should be “We begin”.

• p. 162, last line: \( X_n = 5 \)” should be “\( X_n = -5 \)”.

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• p. 205, Exercise A.4.5: “contraction” should be “contradiction”.

• p. 206, line 4: “g(x)/h(x)” should be “|g(x)/h(x)|”.

• p. 206, line 7: “limsup” should be “lim”.

ERRATA FOR THE ONLINE SOLUTIONS FILE:

[With thanks to Danny Cao and Felix Pahl and Byron Schmuland.]

• Exercise 2.7.14(a): “for all $a \neq b$” should be “for all $\{a, b\} \in \mathcal{J}$”.

• Exercise 2.7.22(a): add a description of $\Omega$, and extend $\mathcal{F}$ to allow for separate subsets $A$ and $B$, not just a single subset $A$.

• Exercise 3.6.6(a): solution is not quite correct (and will be revised).

• Exercise 4.5.10: solution is not correct, and should be replaced by the one at: http://probability.ca/jeff/ftpdir/Ex4.5.10sol.pdf

• Exercise 8.5.20(c): this is not quite correct as stated, since the chain might not be irreducible even on the state spaces $S_r$. But it is still true that there is at least one stationary distribution on each $S_r$, hence at least two stationary distributions in total.

• Exercise 9.5.14(a): answer should be 1, not $\infty$.

• Exercise 13.4.6: solution is incorrect (and will be revised).

• Exercise 13.4.10: solution is incorrect (and will be revised). For a better example, let $P(X = 1, Y = 1) = P(X = 1, Y = -1) = P(X = -1, Y = 2) = P(X = -1, Y = -2) = 1/4$. Then $X$ and $Y$ are clearly not independent, but $E(Y|X) = 0 = E(Y)$.