

**Errata for the SECOND EDITION of “A First Look at Rigorous Probability”,
by Jeffrey S. Rosenthal, World Scientific Publishing Co., 2006.**

(Note: throughout, “line $-x$ ” means x lines from the bottom.)

Errata to Seventh Printing, 2016:

[With thanks to Daehyun Kim.]

- Page 77, lines 11 and 12 (twice): replace the word “obtaining” with the word “reaching” (for greater clarity).

Errata to Sixth Printing, 2013 (to be corrected in Seventh Printing, 2016):

[With thanks to Anthony Brooms, Geoffrey Brown, Julian Ziegler Hunts, Owen Lyne, Byron Schmuland, Adrian Torchiana, Peter Tye, and Martin Yang.]

- Page 3 middle, change “not used elsewhere in this book” to “not required elsewhere in this book”.
- Page 5 middle, change “mathematical rigorous” to “mathematically rigorous”.
- Page 21, line 8: “ $A \subseteq \mathcal{F}$ ” should be “ $A \in \mathcal{F}$ ”.
- Page 23, line -3 : “a algebra” should be “an algebra”.
- Page 29, eqn (3.1.2): “ $X(w)$ ” should be “ $X(\omega)$ ”.
- Page 40, Exercise 3.6.13, line 2: “ P ” should be “ \mathbf{P} ”, and “ X ” should be “ X_n ”, and “for all i ” should be “for all n ”.
- Page 40, Exercise 3.6.18, line 2: better to change “lim” to “lim inf” to avoid issues of the *existence* of the limit. (Alternatively, the current definition of S_x can be taken to mean the limit *exists* and is $\leq x$.)
- Page 46: omit the last two sentences (since we are already assuming that the X_n are non-negative).
- Page 57, line -4 : “ μ_y ” should be “ μ_Y ”.
- Page 87, second half: “Sterling” should be “Stirling” (twice).
- Page 71, Exercise 6.3.1: Replace the first sentence by, “Let μ have density $4x^3 \mathbf{1}_{0 < x < 1}$, and let ν have density $\frac{1}{2}x \mathbf{1}_{0 < x < 2}$.”

- Page 119, lines 1–2: since Y is actually defined as a function on $(0, 1)$, it is better to replace “ $m(Y(n+1) - Y(n)) < \infty$ discontinuities of size $\geq 1/m$ within the interval $(n, n+1]$ ” with “ $m[Y(1 - \frac{1}{n}) - Y(\frac{1}{n})] < \infty$ discontinuities of size $\geq 1/m$ within the interval $(\frac{1}{n}, 1 - \frac{1}{n})$ ”.
- Page 121, Exercise 10.3.2, line 2: omit the comma just before the period.
- Page 134: the last three lines of the proof of Theorem 11.2.2 are poorly phrased, since $\phi_n(t)$ might be complex, and should be replaced by e.g.: “ $n \rightarrow \infty$. This means that for any $\epsilon > 0$, for sufficiently large n we have $|nq_n| \leq \epsilon$. Hence, as $n \rightarrow \infty$ the limit is multiplied by e^{nq_n} , which becomes arbitrarily close to 1 and hence can be ignored.”
- Page 147, lines 17–18: “ μ ” and “ ν ” should be interchanged (twice).
- Page 178, 2/3 of the way down the page: “only if” and “if” should be interchanged.
- Page 181, Exercise 15.2.7: change “a Markov chain” to “an irreducible Markov chain”.
- Page 183, Exercise 15.3.3, line 3: change “ A_0 ” to “ A_1 ”.
- Page 187, line –12: “ $\frac{1}{n} \mathbf{E} \left(Y_{\lfloor \frac{sn}{n} \rfloor}^{(n)} Y_{\lfloor \frac{tn}{n} \rfloor}^{(n)} \right) = \mathbf{E} \left((Z_1 + \dots + Z_{\lfloor sn \rfloor}) (Z_1 + \dots + Z_{\lfloor tn \rfloor}) \right)$ ” should be “ $\mathbf{E} \left(Y_{\lfloor \frac{sn}{n} \rfloor}^{(n)} Y_{\lfloor \frac{tn}{n} \rfloor}^{(n)} \right) = \frac{1}{n} \mathbf{E} \left((Z_1 + \dots + Z_{\lfloor sn \rfloor}) (Z_1 + \dots + Z_{\lfloor tn \rfloor}) \right)$ ”.
- Page 192, Exercise 15.6.8, line 2: change “ $\int_0^t b B_s ds$ ” to “ $\int_0^t b dB_s$ ”.

Errata to Fifth Printing, 2011 (corrected in Sixth Printing, 2013):

[After teaching from this book again after a five-year break, and also with thanks to Daniel Firka, Julian Ziegler Hunts, Jianlin Zou, Luis Mendo Tomás, and Sebastiaan Janssens.]

- Page 19, Exercise 2.5.6: change “finite unions” to “finite disjoint unions”, and note that \mathbf{P} is extended to A_n by finite additivity.
- Page 20: for clarity, replace “distributions” by “measures” (twice).
- Page 22, line 2: change “finite unions” to “finite disjoint unions”.
- Page 25, line 1: “setset” should be “subset”.
- Page 35, line 4: “considering” should be “consider”.
- Page 36, line 9: missing “}”.
- Page 36, several places: omit extra $\{\dots\}$, e.g. “ $\{H_n \cap H_{n+1}\}$ ” should be simply “ $H_n \cap H_{n+1}$ ”, etc.
- Page 39, Exercise 3.6.5: for clarity, assume $\mathcal{F} = 2^\Omega$.

- Page 39, line 4: “P” should be “**P**”.
- Page 40, Exercise 3.6.11: the notation “ \sim ” has not yet been introduced, so “ $X_n \sim \text{Uniform}(\{1, 2, \dots, n\})$ ” should be replaced by “ $\mathbf{P}(X_n = i) = 1/n$ for $i = 1, 2, \dots, n$ ”.
- Page 40, Exercise 3.6.13: should be moved to LATER (since it uses expectation), e.g. as Exercise 4.5.16. (And “E” should be “**E**”, twice.) It could be replaced by e.g.

Let X_1, X_2, \dots be defined jointly on some probability space (Ω, \mathcal{F}, P) , with $\sum_{i=1}^{\infty} i^2 \mathbf{P}(i \leq X < i + 1) \leq C < \infty$ for all i . Prove that $\mathbf{P}[X_n \geq n \text{ i.o.}] = 0$.

- Page 46, statement of Theorem 4.2.2: assume the X_n are non-negative, and then omit “ $\mathbf{E}(X_1) > -\infty$ ” (since we haven’t yet defined expected values of general random variables).
- Page 46, first line of proof: for greater clarity, replace “(3.1.6)” by “Proposition 3.1.5.(iii)”. (Similarly page 107, line 5.)
- Page 46, lines -3 and -2: “E” should be boldface (twice).
- Page 49, exercise 4.3.3(a): change “ Z^+ and Z^- ” to “ $Z^+ - Z^-$ ”.
- Page 54, Exercise 4.5.13(d): replace “ $\mathbf{E}(X) < \infty$ ” by “ $0 < \mathbf{E}(X) < \infty$ ”.
- Page 58, Lemma 5.2.1: note that the converse also holds.
- Page 60: introduce the abbreviations “WLLN” and “SLLN”.
- Page 65, Exercise 5.5.9, Hint: specify that $y > 0$ for the first part, too.
- Page 66, Exercise 5.5.13, Hint: “ r different sums” should be “ $r + 1$ different sums”.
- Page 71, Exercise 6.3.1 is a repeat of Exercise 4.5.1 (page 52), and should be replaced by e.g.:

Let μ have density $x^3 \mathbf{1}_{0 < x < 1}$, and let ν have density $x \mathbf{1}_{0 < x < 2}$.

- Compute $\mathbf{E}(X)$ where $\mathcal{L}(X) = \frac{1}{3}\mu + \frac{2}{3}\nu$.
- Compute $\mathbf{E}(Y^2)$ where $\mathcal{L}(Y) = \frac{1}{6}\mu + \frac{1}{3}\delta_2 + \frac{1}{2}\delta_5$.
- Compute $\mathbf{E}(Z^3)$ where $\mathcal{L}(Z) = \frac{1}{8}\mu + \frac{1}{8}\nu + \frac{1}{4}\delta_3 + \frac{1}{2}\delta_4$.

- Page 76, Exercise 7.2.5(d): for clarity, prepend “Use the fact that $s(c) = 1$ to”.
- Page 85, statement of Theorem 8.1.1, displayed eqn: “ nu_{i_0} ” should be “ ν_{i_0} ”.
- Page 114, line -1: “ β^α ” should be “ $\beta^{-\alpha}$ ”.
- Page 115, line 1: “ t^{x-1} ” should be “ $t^{\alpha-1}$ ”.
- Page 117, line 2: for clarity, change “distributions” to “measures”.

- Page 125, middle of page, displayed equations: final inequality (\leq) is actually an equality ($=$).
- Page 144, line -5 : omit the second “then”.
- Page 159, Exercise 13.4.5: Assume $\mathbf{P}\{1\} = \mathbf{P}\{2\} = \mathbf{P}\{3\} = 1/3$.
- Page 159, Exercise 13.4.6: Change “ $\mathbf{E}(X | \mathcal{G})$ and $\mathbf{E}(Y | \mathcal{G})$ need not be independent” to “we might have $\mathbf{E}(XY | \mathcal{G}) \neq \mathbf{E}(X | \mathcal{G}) \mathbf{E}(Y | \mathcal{G})$ ”.
- Page 165, line 3: “Theorem 14.1.3” should be “Theorem 14.1.5”.
- Page 171: for clarity, replace “for a submartingale, the same inequality holds” by “for submartingales, similarly inequalities hold”.
- Page 172, line -2 : “Then follows” should be “It then follows”.
- Page 173, Exercise 14.4.5: Assume $\mathbf{E}|Z| < \infty$.
- Page 181, Theorem 15.2.3: “ $\rightarrow 0$ ” should be “ $= 0$ ”.
- Page 187: $O(1/n)$ should be $O(1/\sqrt{n})$ (three times).
- Page 192: Exercise 15.6.7: For greater generality, replace “ $X_t = at + bB_t$ ” by “ $X_t = X_0 + at + bB_t$ ” in part (a), and “ $N(at, b^2t)$ ” by “ $N(X_0 + at, b^2t)$ ” in part (b).
- Page 192: Exercise 15.6.8: Replace “ $\int_0^t s ds$ ” by “ $\int_0^t a ds$ ”.
- Page 206 middle, “it suffice to assume” should be “it suffices to assume”.

Errata to Fourth Printing, 2010 (corrected in Fifth Printing, 2011):

[With thanks to David Alexander, Martin Hazelton, Andrea Lecchini-Visintini, Gareth Roberts, Igal Sason, Mohsen Soltanifar, and Albert Zevelev.]

- p. 13, line 3 of proof of Lemma 2.3.11: “ \mathcal{B}_m^C ” should be “ B_m^C ”.
- p. 23, Exercise 2.7.3, part (a): “an semialgebra” should be “a semialgebra”.
- p. 31, line 5: “ $f^{-1}((\infty, x])$ ” should be “ $f^{-1}((-\infty, x])$ ”.
- p. 44, line 1, and again on line 19: “ $\sum_j y_j \mathbf{1}_{B_i}$ ” should be “ $\sum_j y_j \mathbf{1}_{B_j}$ ”.
- p. 48, line -5 : “non non-negative” should be “not non-negative”.
- p. 66, line -8 : “Holder” should be “Jensen”.
- p. 69, first line of Proposition 6.2.1: “ μ_I ” should be “ μ_i ”.
- p. 74, line 7: “ $F(z)$ for each $z \in \mathbf{R}$ ” should be “ $F(x)$ for each $x \in \mathbf{R}$ ”.

- p. 85, displayed equation in Theorem 8.1.1: “ $X_{=i_1}$ ” should be “ $X_1 = i_1$ ”.
- p. 106, line 15: “integral” should be “integrable”.
- p. 106, line 17: “ $\mathbf{E}(X)$ ” should be “ $\mathbf{E}(X_0)$ ”.
- p. 106, line –1: “ $E(Y)$ ” should be “ $\mathbf{E}(Y)$ ”.
- p. 136, Exercise 11.3.2: “ Y_n ” should be “ Y_k ” (twice).
- p. 137, conclusion of Theorem 11.4.1: “ $\mu \Rightarrow \mu$ ” should be “ $\mu_n \Rightarrow \mu$ ”.
- p. 146, line –6: “ μ_{sing} ” should be “ μ_{sing} ”.
- p. 155, middle: “ $\{\{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbf{R}\}$ ” should be “ $\sigma(\{\{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbf{R}\})$ ”.
- p. 171, line 11: “ $\mathbf{E} \left(\lim_{M \rightarrow \infty} U_M^{(\alpha, \beta)} = \infty \right) = \infty$ ” should be “ $\mathbf{E} \left(\lim_{M \rightarrow \infty} U_M^{(\alpha, \beta)} \right) = \infty$ ”.
- p. 178, line –6: “ $\{X_{t_1}, \dots, X_{t_k}\} \in H$ ” should be “ $\{(X_{t_1}, \dots, X_{t_k}) \in H\}$ ”.
- p. 192, Exercise 15.6.8: replace “ $Z_t = \exp[-(a + \frac{1}{2}b^2)t + X_t]$ ” with “ $Z_t = \exp[-2aX_t/b^2]$ ”. (In fact, the first Z_t is also a martingale, but it is less useful than the second version.)
- p. 202, lines 14–15: to avoid subtleties about equivalent repeating decimals, perhaps replace “ $c_i = 4$ if $d_i = 5$, while $c_i = 5$ if $d_i \neq 5$ ” with “ $c_i = 2$ if $d_i \geq 5$, while $c_i = 7$ if $d_i < 5$ ”. (This is not strictly necessary, but it makes the argument a bit cleaner.)
- p. 204, Exercise A.3.8: change “ $\sum_{i=1}^{\infty}$ ” to “ $\sum_{i=2}^{\infty}$ ” (twice), and “ \int_1^{∞} ” to “ \int_2^{∞} ”.
- p. 204, Exercise A.3.9: change “ $\sum_{i=1}^{\infty}$ ” to “ $\sum_{i=3}^{\infty}$ ” (twice).
- p. 207, second line of Exercise A.5.1: “equivalence class” should be “equivalence relation”.

Errata to Second Printing, 2007 (corrected in Fourth Printing, 2010):

[With thanks to Orn Arnaldsson, Bent Jørgensen, Chris Mansley, Kohei Nagamachi, Patrick Rabau, Mohsen Soltanifar, Hermann Thorisson.]

- p. 19, Exercise 2.5.6, and also p. 22, proof of Lemma 2.6.2: replace “ $A_1, A_2, \dots \in \mathcal{J}$ ” by “ A_1, A_2, \dots are finite unions of elements of \mathcal{J} ”.
- p. 23, Exercise 2.6.4: “ $\mathbf{P}(\emptyset) = 1$ ” should be “ $\mathbf{P}(\Omega) = 1$ ”.
- p. 23, Exercise 2.7.3, part (b): interchange “semialgebra” and “algebra”. (Also, for stylistic improvement, swap parts (a) and (b).)

- p. 30, last line of proof of Proposition 3.1.5: “ $\{X \leq x\}$ ” should be “ $\{Z \leq x\}$ ”.
- p. 33, line 7: second “ $\mathbf{P}(X \in T)$ ” should be “ $\mathbf{P}(Y \in S)$ ”.
- p. 39, Exercise 3.6.8: exercise is correct, but special cases like “ $d \leq b + c - a$ ” and “ $d > b + c - a$ ” should be modified.
- p. 40, Exercise 3.6.14: insert “independent” before “non-negative”.
- p. 74, the proof of Lemma 7.1.2 is sloppily written and should be replaced by:

Since F is right-continuous, we have that $\inf\{x; F(x) \geq u\} = \min\{x; F(x) \geq u\}$, i.e. the infimum is actually obtained. It follows that $\phi(u) \leq x$ if and only if $u \leq F(x)$. Hence, since $0 \leq F(x) \leq 1$, we obtain that $\mathbf{P}(\phi(U) \leq x) = \mathbf{P}(U \leq F(x)) = F(x)$.
- pp. 113–114: “Example 9.5.9” should be “Exercise 9.5.9”, and similarly for 9.5.11 and 9.5.12.
- p. 114, Exercise 9.5.12: “characteristic function” should be “moment generating function”.
- p. 118 middle, the three lines following Figure 10.1.2: replace “ g ” by “ f ” (five times).
- p. 118, line –5: “ $F(w) \geq b$ ” should be “ $F(z) \geq b$ ”.
- p. 121, Exercise 10.3.5: “four conditions” should be “five conditions”.
- p. 126, statement of Lemma 11.1.2: “ $\phi(t)$ ” should be “ e^{itx} ”.
- p. 128, line –7: “ π ” should be removed from the equation.
- p. 131, line 9: “ \lim_n ” should be “ \lim_k ” (twice); “ F_n ” should be “ F_{n_k} ” (twice); and “ μ_n ” should be “ μ_{n_k} ”.
- p. 136, lines 3–4: the sentence in brackets is somewhat misleading and should be revised or omitted.
- p. 136, eqn (11.3.1): “ $\mathbf{1}_{|Z_{n_k}| \geq \epsilon s_n}$ ” should be “ $\mathbf{1}_{|Z_{n_k}| \geq \epsilon s_n}$ ”
- p. 166, lines 6–7: “bets \$1 on tails, then if they win they bet \$2 on heads” should be “bets \$1 on heads, then if they win they bet \$2 on tails”.
- p. 173, Exercise 14.4.1: should say “ $\mathbf{P}(Z_i = 1) = \mathbf{P}(Z_i = 0) = 1/2$ ”, and “ $X_1 = 2Z_1 - 1$ ”.
- p. 174, Exercise 14.4.12(a), Hint: “ $\mathbf{P}(\tau \geq 3m)$ ” should be “ $\mathbf{P}(\tau > 3m)$ ”.

Errata to First Printing, 2006 (corrected in Second Printing, 2007):

[With thanks to Joe Blitzstein, Saad Siddiqui, Emil Zeuthen.]

- p. 9, line –10: “*all* intervals” should be “*all* subsets”.
- p. 18, eqn. (2.5.2): “ $P(B)$ ” should be “ $\mathbf{P}(B)$ ”.
- p. 19, the last sentence in the proof of corollary 2.5.4 is questionable (since we may have $D_n \notin \mathcal{J}$), and should be replaced by:

It then follows from (2.5.5) that

$$\mathbf{P}\left(\bigcup_n B_n\right) = \mathbf{P}\left(\bigcup_n D_n\right) = \mathbf{P}\left(\bigcup_n \bigcup_{i=1}^{k_n} J_{ni}\right) = \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}).$$

On the other hand,

$$B_n = \bigcup_{m \leq n} \bigcup_{i=1}^{k_m} (J_{mi} \cap B_n)$$

and the union is disjoint, with $J_{ni} \subseteq B_n$, so

$$\mathbf{P}(B_n) = \sum_{m \leq n} \sum_{i=1}^{k_m} \mathbf{P}(J_{mi} \cap B_n) \geq \sum_{i=1}^{k_n} \mathbf{P}(J_{ni} \cap B_n) = \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}),$$

and hence

$$\sum_n \mathbf{P}(B_n) \geq \sum_n \sum_{i=1}^{k_n} \mathbf{P}(J_{ni}) = \mathbf{P}\left(\bigcup_n B_n\right).$$

- p. 20, eqn. (2.5.10): “ $(\infty, x]$ ” should be “ $(-\infty, x]$ ”.
- p. 22, line –10: “ P_1 ” should be “ \mathbf{P}_1 ”, and “ P_2 ” should be “ \mathbf{P}_2 ”.
- p. 151, first line of Section 13.1: “We being” should be “We begin”.
- p. 162, last line: “ $X_n = 5$ ” should be “ $X_n = -5$ ”.
- p. 205, Exercise A.4.5: “contraction” should be “contradiction”.
- p. 206, line 4: “ $g(x)/h(x)$ ” should be “ $|g(x)/h(x)|$ ”.
- p. 206, line 7: “limsup” should be “lim”.

ERRATA FOR THE ONLINE SOLUTIONS FILE:

[With thanks to Danny Cao and Felix Pahl and Byron Schmuland.]

- Exercise 2.7.14(a): “for all $a \neq b$ ” should be “for all $\{a, b\} \in \mathcal{J}$ ”.
- Exercise 2.7.22(a): add a description of Ω , and extend \mathcal{F} to allow for separate subsets A and B , not just a single subset A .
- Exercise 3.6.6(a): solution is not quite correct (and will be revised).
- Exercise 4.5.10: solution is not correct, and should be replaced by the one at:
<http://probability.ca/jeff/ftpdirex4.5.10sol.pdf>
- Exercise 8.5.20(c): this is not quite correct as stated, since the chain might not be irreducible even on the state spaces S_r . But it is still true that there is at least one stationary distribution on each S_r , hence at least two stationary distributions in total.
- Exercise 9.5.14(a): answer should be 1, not ∞ .
- Exercise 13.4.6: solution is incorrect (and will be revised).
- Exercise 13.4.10: solution is incorrect (and will be revised). For a better example, let $\mathbf{P}(X = 1, Y = 1) = \mathbf{P}(X = 1, Y = -1) = \mathbf{P}(X = -1, Y = 2) = \mathbf{P}(X = -1, Y = -2) = 1/4$. Then X and Y are clearly not independent, but $\mathbf{E}(Y|X) = 0 = \mathbf{E}(Y)$.