Errata for the SECOND EDITION of “A First Look at Rigorous Probability”,

(Note: throughout, “line −x” means x lines from the bottom.)

Errata to Seventh Printing, 2016:

[With thanks to Daehyun Kim.]

• Page 77, lines 11 and 12 (twice): replace the word “obtaining” with the word “reaching” (for greater clarity).

Errata to Sixth Printing, 2013 (to be corrected in Seventh Printing, 2016):

[With thanks to Anthony Brooms, Geoffrey Brown, Julian Ziegler Hunts, Owen Lyne, Byron Schmuland, Adrian Torchiana, Peter Tye, and Martin Yang.]

• Page 3 middle, change “not used elsewhere in this book” to “not required elsewhere in this book”.

• Page 5 middle, change “mathematical rigorous” to “mathematically rigorous”.

• Page 21, line 8: “A ⊆ F” should be “A ∈ F”.

• Page 23, line −3: “a algebra” should be “an algebra”.

• Page 29, eqn (3.1.2): “X(w)” should be “X(ω)”.

• Page 40, Exercise 3.6.13, line 2: “P” should be “P”, and “X” should be “X_n”, and “for all i” should be “for all n”.

• Page 40, Exercise 3.6.18, line 2: better to change “lim” to “lim inf” to avoid issues of the existence of the limit. (Alternatively, the current definition of S_x can be taken to mean the limit exists and is ≤ x.)

• Page 46: omit the last two sentences (since we are already assuming that the X_n are non-negative).

• Page 57, line −4: “µ_y” should be “µ_Y”.

• Page 87, second half: “Sterling” should be “Stirling” (twice).

• Page 71, Exercise 6.3.1: Replace the first sentence by, “Let µ have density 4x^31_{0<x<1}, and let ν have density \frac{1}{2}x1_{0<x<2}.”
• Page 119, lines 1–2: since Y is actually defined as a function on (0, 1), it is better to replace “m(Y(n + 1) − Y(n)) < ∞ discontinuities of size ≥ 1/m within the interval (n, n + 1)” with “m[Y(1 − 1/n) − Y(1/n)] < ∞ discontinuities of size ≥ 1/m within the interval (1/n, 1 − 1/n)”.

• Page 121, Exercise 10.3.2, line 2: omit the comma just before the period.

• Page 134: the last three lines of the proof of Theorem 11.2.2 are poorly phrased, since ϕ_n(t) might be complex, and should be replaced by e.g.: “n → ∞. This means that for any ε > 0, for sufficiently large n we have |nq_n| ≤ ε. Hence, as n → ∞ the limit is multiplied by e^{nq_n}, which becomes arbitrarily close to 1 and hence can be ignored.”

• Page 147, lines 17–18: “μ” and “ν” should be interchanged (twice).

• Page 178, 2/3 of the way down the page: “only if” and “if” should be interchanged.

• Page 181, Exercise 15.2.7: change “a Markov chain” to “an irreducible Markov chain”.

• Page 183, Exercise 15.3.3, line 3: change “A_0” to “A_1”.

• Page 187, line −12: “1/n E \left( Y_{[sn]}^{(n)} Y_{[tn]}^{(n)} \right) = E \left( (Z_1 + \ldots + Z_{[sn]}) (Z_1 + \ldots + Z_{[tn]} \right)” should be “1/n E \left( Y_{[sn]}^{(n)} Y_{[tn]}^{(n)} \right) = E \left( (Z_1 + \ldots + Z_{[sn]}) (Z_1 + \ldots + Z_{[tn]} \right)”.

• Page 192, Exercise 15.6.8, line 2: change “\int_0^t b B_s ds” to “\int_0^t b dB_s”.

Errata to Fifth Printing, 2011 (corrected in Sixth Printing, 2013):

[After teaching from this book again after a five-year break, and also with thanks to Daniel Firka, Julian Zieglers Hunts, Jianlin Zou, Luis Mendo Tomses, and Sebastiaan Janssens.]

• Page 19, Exercise 2.5.6: change “finite unions” to “finite disjoint unions”, and note that P is extended to A_n by finite additivity.

• Page 20: for clarity, replace “distributions” by “measures” (twice).

• Page 22, line 2: change “finite unions” to “finite disjoint unions”.

• Page 25, line 1: “setset” should be “subset”.

• Page 36, line 4: “considering” should be “consider”.

• Page 36, line 9: missing “}”.

• Page 36, several places: omit extra {...}, e.g. “\{H_n \cap H_{n+1}\}” should be simply “H_n \cap H_{n+1}”, etc.

• Page 39, Exercise 3.6.5: for clarity, assume \mathcal{F} = 2^\Omega.
• Page 39, line 4: “P” should be “P”.

• Page 40, Exercise 3.6.11: the notation “∼” has not yet been introduced, so “\(X_n \sim \text{Uniform}\{\{1,2,\ldots,n\}\}” should be replaced by “\(P(X_n = i) = 1/n\) for \(i = 1,2,\ldots,n\)”.

• Page 40, Exercise 3.6.13: should be moved to LATER (since it uses expectation), e.g. as Exercise 4.5.16. (And “E” should be “E”, twice.) It could be replaced by e.g.

\[
\text{Let } X_1,X_2,\ldots \text{ be defined jointly on some probability space } (\Omega,\mathcal{F},P), \text{ with } \sum_{i=1}^{\infty} i^2 P(i \leq X < i + 1) \leq C < \infty \text{ for all } i. \text{ Prove that } P[X_n \geq n \text{ i.o.}] = 0.
\]

• Page 46, statement of Theorem 4.2.2: assume the \(X_n\) are non-negative, and then omit “\(E(X_1) > -\infty\)” (since we haven’t yet defined expected values of general random variables).

• Page 46, first line of proof: for greater clarity, replace “\((3.1.6)\)” by “Proposition 3.1.5.(iii)” (Similarly page 107, line 5.)

• Page 46, lines -3 and -2: “E” should be boldface (twice).

• Page 49, exercise 4.3.3(a): change “\(Z^+ \text{ and } Z^-\)” to “\(Z^+ - Z^-\)”.

• Page 54, Exercise 4.5.13(d): replace “\(E(X) < \infty\)” by “\(0 < E(X) < \infty\)”.

• Page 58, Lemma 5.2.1: note that the converse also holds.

• Page 60: introduce the abbreviations “WLLN” and “SLLN”.

• Page 65, Exercise 5.5.9, Hint: specify that \(y > 0\) for the first part, too.

• Page 66, Exercise 5.5.13, Hint: “\(r\) different sums” should be “\(r + 1\) different sums”.

• Page 71, Exercise 6.3.1 is a repeat of Exercise 4.5.1 (page 52), and should be replaced by e.g.:

Let \(\mu\) have density \(x^31_{0<x<1}\), and let \(\nu\) have density \(x1_{0<x<2}\).

(a) Compute \(E(X)\) where \(L(X) = \frac{1}{3}\mu + \frac{2}{3}\nu\).

(b) Compute \(E(Y^2)\) where \(L(Y) = \frac{1}{5}\mu + \frac{3}{5}\delta_2 + \frac{1}{2}\delta_5\).

(c) Compute \(E(Z^3)\) where \(L(Z) = \frac{2}{5}\mu + \frac{1}{5}\nu + \frac{1}{4}\delta_3 + \frac{1}{2}\delta_4\).

• Page 76, Exercise 7.2.5(d): for clarity, prepend “Use the fact that \(s(c) = 1\) to”.

• Page 85, statement of Theorem 8.1.1, displayed eqn: “\(nu_{i_0}\)” should be “\(\nu_{i_0}\)”.

• Page 114, line -1: “\(\beta^0\)” should be “\(\beta^{-\alpha}\)”.

• Page 115, line 1: “\(\ell x^{-1}\)” should be “\(\ell^{\alpha-1}\)”.

• Page 117, line 2: for clarity, change “distributions” to “measures”.

3
• Page 125, middle of page, displayed equations: final inequality (≤) is actually an equality (=).

• Page 144, line −5: omit the second “then”.

• Page 159, Exercise 13.4.5: Assume \( P\{1\} = P\{2\} = P\{3\} = 1/3 \).

• Page 159, Exercise 13.4.6: Change “\( E(X \mid G) \) and \( E(Y \mid G) \) need not be independent” to “we might have \( E(XY \mid G) \neq E(X \mid G) \ E(Y \mid G) \)”.

• Page 165, line 3: “Theorem 14.1.3” should be “Theorem 14.1.5”.

• Page 171: for clarity, replace “for a submartingale, the same inequality holds” by “for submartingales, similarly inequalities hold”.

• Page 172, line −2: “Then follows” should be “It then follows”.

• Page 173, Exercise 14.4.5: Assume \( E[Z] < \infty \).

• Page 181, Theorem 15.2.3: “\( \rightarrow 0 \)” should be “\( = 0 \)”.

• Page 187: \( O(1/n) \) should be \( O(1/\sqrt{n}) \) (three times).

• Page 192: Exercise 15.6.7: For greater generality, replace “\( X_t = at + bB_t \)” by “\( X_t = X_0 + at + bB_t \)” in part (a), and “\( N(at, b^2 t) \)” by “\( N(X_0 + at, b^2 t) \)” in part (b).

• Page 192: Exercise 15.6.8: Replace “\( \int_0^t s \, ds \)” by “\( \int_0^t a \, ds \)”.

• Page 206 middle, “it suffice to assume” should be “it suffices to assume”.

Errata to Fourth Printing, 2010 (corrected in Fifth Printing, 2011):

[With thanks to David Alexander, Martin Hazelton, Andrea Lecchini-Visintini, Gareth Roberts, Igal Sason, Mohsen Soltanifar, and Albert Zevelev.]

• p. 13, line 3 of proof of Lemma 2.3.11: “\( B_m^C \)” should be “\( B_m^C \)”.

• p. 23, Exercise 2.7.3, part (a): “an semialgebra” should be “a semialgebra”.

• p. 31, line 5: “\( f^{-1}((\infty, x]) \)” should be “\( f^{-1}((\infty, x]) \)”.

• p. 44, line 1, and again on line 19: “\( \sum_j y_j \mathbf{1}_{B_j} \)” should be “\( \sum_j y_j \mathbf{1}_{B_j} \)”.

• p. 48, line −5: “non non-negative” should be “not non-negative”.

• p. 66, line −8: “Holder” should be “Jensen”.

• p. 69, first line of Proposition 6.2.1: “\( \mu_I \)” should be “\( \mu_i \)”.

• p. 74, line 7: “\( F(z) \) for each \( z \in \mathbb{R} \)” should be “\( F(x) \) for each \( x \in \mathbb{R} \)”.

4
• p. 85, displayed equation in Theorem 8.1.1: “$X = i_1$” should be “$X_1 = i_1$”.

• p. 106, line 15: “integral” should be “integrable”.

• p. 106, line 17: “$\mathbf{E}(X)$” should be “$\mathbf{E}(X_0)$”.

• p. 106, line −1: “$\mathbf{E}(Y)$” should be “$\mathbf{E}(Y)$”.

• p. 136, Exercise 11.3.2: “$Y_n$” should be “$Y_k$” (twice).

• p. 137, conclusion of Theorem 11.4.1: “$\mu \Rightarrow \mu$” should be “$\mu_n \Rightarrow \mu$”.

• p. 146, line −6: “$\mu_{sing}$” should be “$\mu_{sing}$”.

• p. 155, middle: “$\{\{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbb{R}\}$” should be “$\sigma(\{\{X_1 \leq a\} \cap \{X_2 \leq b\} : a, b \in \mathbb{R}\})$”.

• p. 171, line 11: “$\mathbf{E}\left(\lim_{M \to \infty} U_M^{(\alpha,\beta)} = \infty\right) = \infty$” should be “$\mathbf{E}\left(\lim_{M \to \infty} U_M^{(\alpha,\beta)}\right) = \infty$”.

• p. 178, line −6: “$\{X_{t_1}, \ldots, X_{t_k}\} \in H\}$” should be “$\{(X_{t_1}, \ldots, X_{t_k}) \in H\}$”.

• p. 192, Exercise 15.6.8: replace “$Z_t = \exp[-(a+\frac{1}{2}b^2)t+X_t]$” with “$Z_t = \exp[-2aX_t/b^2]$”.

• p. 202, lines 14–15: to avoid subtleties about equivalent repeating decimals, perhaps replace “$c_i = 4$ if $d_i = 5$, while $c_i = 5$ if $d_i \neq 5$” with “$c_i = 2$ if $d_i \geq 5$, while $c_i = 7$ if $d_i < 5$”.

• p. 204, Exercise A.3.8: change “$\sum_{i=1}^{\infty}$” to “$\sum_{i=2}^{\infty}$” (twice), and “$\int_1^{\infty}$” to “$\int_2^{\infty}$”.

• p. 204, Exercise A.3.9: change “$\sum_{i=1}^{\infty}$” to “$\sum_{i=3}^{\infty}$” (twice).

• p. 207, second line of Exercise A.5.1: “equivalence class” should be “equivalence relation”.

Errata to Second Printing, 2007 (corrected in Fourth Printing, 2010):

[With thanks to Orn Arnaldsson, Bent Jørgensen, Chris Mansley, Kohei Nagamachi, Patrick Rabau, Mohsen Soltanifar, Hermann Thorisson.]

• p. 19, Exercise 2.5.6, and also p. 22, proof of Lemma 2.6.2: replace “$A_1, A_2, \ldots \in \mathcal{J}$” by “$A_1, A_2, \ldots$ are finite unions of elements of $\mathcal{J}$”.

• p. 23, Exercise 2.6.4: “$\mathbf{P}(\emptyset) = 1$” should be “$\mathbf{P}(\Omega) = 1$”.

• p. 23, Exercise 2.7.3, part (b): interchange “semialgebra” and “algebra”. (Also, for stylistic improvement, swap parts (a) and (b).)
• p. 30, last line of proof of Proposition 3.1.5: \( \{X \leq x\} \) should be \( \{Z \leq x\} \).

• p. 33, line 7: second \( \mathbf{P}(X \in T) \) should be \( \mathbf{P}(Y \in S) \).

• p. 39, Exercise 3.6.8: exercise is correct, but special cases like \( d \leq b + c - a \) and \( d > b + c - a \) should be modified.

• p. 40, Exercise 3.6.14: insert “independent” before “non-negative”.

• p. 74, the proof of Lemma 7.1.2 is sloppily written and should be replaced by:

Since \( F \) is right-continuous, we have that \( \inf \{ x; F(x) \geq u \} = \min \{ x; F(x) \geq u \} \), i.e. the infimum is actually obtained. It follows that \( \phi(u) \leq x \) if and only if \( u \leq F(x) \). Hence, since \( 0 \leq F(x) \leq 1 \), we obtain that
\[
\mathbf{P}(\phi(U) \leq x) = \mathbf{P}(U \leq F(x)) = F(x).
\]

• pp. 113–114: “Example 9.5.9” should be “Exercise 9.5.9”, and similarly for 9.5.11 and 9.5.12.

• p. 114, Exercise 9.5.12: “characteristic function” should be “moment generating function”.

• p. 118 middle, the three lines following Figure 10.1.2: replace “\( g \)” by “\( f \)” (five times).

• p. 118, line –5: \( F(w) \geq b \) should be \( F(z) \geq b \).

• p. 121, Exercise 10.3.5: “four conditions” should be “five conditions”.

• p. 126, statement of Lemma 11.1.2: “\( \phi(t) \)” should be “\( e^{itx} \)”.

• p. 128, line –7: “\( \pi \)” should be removed from the equation.

• p. 131, line 9: “\( \lim \)” should be “\( \lim_k \)” (twice); “\( F_n \)” should be “\( F_{n_k} \)” (twice); and “\( \mu_n \)” should be “\( \mu_{n_k} \)”.

• p. 136, lines 3–4: the sentence in brackets is somewhat misleading and should be revised or omitted.

• p. 136, eqn (11.3.1): \( 1_{\{Z_n \geq s_n \}} \) should be “\( 1_{\{Z_{n_k} \geq s_n \}} \)”.

• p. 166, lines 6–7: “bets $1$ on tails, then if they win they bet $2$ on heads” should be “bets $1$ on heads, then if they win they bet $2$ on tails”.

• p. 173, Exercise 14.4.1: should say \( \mathbf{P}(Z_i = 1) = \mathbf{P}(Z_i = 0) = 1/2 \), and \( X_1 = 2Z_1 - 1 \).

• p. 174, Exercise 14.4.12(a), Hint: \( \mathbf{P}(\tau \geq 3m) \)” should be \( \mathbf{P}(\tau > 3m) \).

p. 9, line −10: “all intervals” should be “all subsets”.

p. 18, eqn. (2.5.2): “P(B)” should be “P(B)”.

p. 19, the last sentence in the proof of corollary 2.5.4 is questionable (since we may have \( D_n \notin J \)), and should be replaced by:

It then follows from (2.5.5) that

\[
P\left( \bigcup_n B_n \right) = P\left( \bigcup_n D_n \right) = P\left( \bigcup_n \bigcup_{i=1}^{k_n} J_{ni} \right) = \sum_n \sum_{i=1}^{k_n} P(J_{ni}) .
\]

On the other hand, \( B_n = \bigcup_{m \leq n} \bigcup_{i=1}^{k_m} (J_{mi} \cap B_n) \) and the union is disjoint, with \( J_{ni} \subseteq B_n \), so

\[
P(B_n) = \sum_{m \leq n} \sum_{i=1}^{k_m} P(J_{mi} \cap B_n) \geq \sum_{i=1}^{k_n} P(J_{ni} \cap B_n) = \sum_{i=1}^{k_n} P(J_{ni}) ,
\]

and hence

\[
\sum_n P(B_n) \geq \sum_{n} \sum_{i=1}^{k_n} P(J_{ni}) = P\left( \bigcup_n B_n \right) .
\]

p. 20, eqn. (2.5.10): “(∞, x]” should be “(−∞, x]”.

p. 22, line −10: “\( P_1 \)” should be “\( P_1 \)”, and “\( P_2 \)” should be “\( P_2 \)”.

p. 151, first line of Section 13.1: “We being” should be “We begin”.

p. 162, last line: “\( X_n = 5 \)” should be “\( X_n = -5 \)”.

p. 205, Exercise A.4.5: “contraction” should be “contradiction”.

p. 206, line 4: “\( g(x)/h(x) \)” should be “\(|g(x)/h(x)|\)”.

p. 206, line 7: “limsup” should be “lim”.

ERRATA FOR THE ONLINE SOLUTIONS FILE:
• Exercise 2.7.14(a): “for all $a \neq b$” should be “for all \{a, b\} $\in \mathcal{J}$”.

• Exercise 2.7.22(a): add a description of $\Omega$, and extend $\mathcal{F}$ to allow for separate subsets $A$ and $B$, not just a single subset $A$.

• Exercise 3.6.6(a): solution is not quite correct (and will be revised).

• Exercise 4.5.10: solution is not correct, and should be replaced by the one at: http://probability.ca/jeff/ftpdir/Ex4.5.10sol.pdf

• Exercise 8.5.20(c): this is not quite correct as stated, since the chain might not be irreducible even on the state spaces $S_r$. But it is still true that there is at least one stationary distribution on each $S_r$, hence at least two stationary distributions in total.

• Exercise 9.5.14(a): answer should be 1, not $\infty$.

• Exercise 13.4.6: solution is incorrect (and will be revised).

• Exercise 13.4.10: solution is incorrect (and will be revised). For a better example, let $P(X = 1, Y = 1) = P(X = 1, Y = -1) = P(X = -1, Y = 2) = P(X = -1, Y = -2) = 1/4$. Then $X$ and $Y$ are clearly not independent, but $E(Y|X) = 0 \neq E(Y)$. 