OPEN PROBLEM!
PRIZE: FREE DINNER!!

As a function of N and a, how large does k need to be so that the sum

$$\sum_{\lambda_1<\lambda_2<\ldots<\lambda_N} \left(\sum_{\substack{j=1 \\ \lambda_j \le a}}^{N} (-1)^j \frac{\binom{-\lambda_j-a+N-2}{N-2-2a}}{\prod_{r=j+1}^{N} (\lambda_r - \lambda_j) \prod_{r=1}^{j-1} (\lambda_j - \lambda_r)} \right)^{2k} \left(\prod_{1\le r<s\le N} (\lambda_s - \lambda_r) \right)^2$$

is finite? Here the sum is taken over all N-tuples of (positive or negative) integers $(\lambda_1, \lambda_2, \ldots, \lambda_N)$ satisfying $\lambda_1 < \lambda_2 < \ldots < \lambda_N$. Also N is a (large) positive integer, a is an integer between 0 and $(N-1)/2$, and $\binom{-\lambda_j-a+N-2}{N-2-2a}$ is a binomial coefficient.

PARTIAL RESULT: k needs to be *at least* as large as $(N^2 - N + 1)/2(a+1)$.

Any other partial results *of any kind* would be appreciated!!

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(By the way, this problem comes from analyzing random walks on the unitary group $U(N)$ iterated k times, and seeing whether they converge in L^2 to Haar measure.)

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