STA410/2102 (Statistical Computation), Fall 2007

Homework #2

Due: In class by 6:10 p.m. **sharp** on Tuesday October 16. (If you prefer, you may bring your assignment to the instructor's office, Sidney Smith Hall room 6024, any time before it is due; slide it under the door if he is not in.)

Warning: Late homeworks, even by one minute, will be penalised!

Reminder: There will be an in-class test on Tuesday October 23, room to be announced.

Note: Homework assignments are to be solved by each student individually. You may discuss assignments in general terms with other students, but you must solve it on your own, including doing all of your own computing and writing.

Include at the top of the first page: Your name and student number.

Special Note: When writing programs in R for homework, you should include some comment lines to explain what you are doing. Also, you should hand in both the program itself, and the program's output.

The assignment:

1. Consider the function $f(x) = x^2 \exp(-(x + \sin(x)))$, with $g(x) \equiv f'(x) = 2\,x \exp(-(x + \sin(x))) + x^2 \exp(-(x + \sin(x)))(-1 - \cos(x))$, as in the file "Riter".

(a) Write a program in R which uses the Secant Method to numerically solve the equation $g(x) = 0$, i.e. to find a root of g. (See the Special Note, above.) [You may wish to begin with the file "Rnewt" from the course web page, but you will need to modify it. For example, the function "gp" should not appear in your program. Also, your program should terminate whenever $x_{n+1} = x_n$.]

(b) Run your program for 100 iterations (or until termination), with initial values x_1 and x_2 equal to the last two digits of your student number. (If those two digits coincide, replace the second one by 10. And, if one of those two digits is 0, replace it by 1 or 2.)

(c) Display a graph of the resulting sequence of 100 values.

(d) Do these values seem to converge to a root of g? Why or why not? Discuss.

(e) Find NON-INTEGER initial values x_1 and x_2 which make the iterations converge to each of (i) 0, (ii) a root near 4, and (iii) a root near 10.

2. Let $X_1, \ldots, X_n$ be i.i.d. with distribution Exponential(λ), having density $\lambda e^{-\lambda x}$, with λ unknown. Suppose we observe $n = 3$ values: $x_1 = 5$, $x_2 = 8$, and $x_3 = 10$.

(a) Write down the likelihood function for λ.

(b) Write down the log-likelihood function for λ.

(c) Write a program in R that uses the Bisection Method to find the MLE $\hat{\lambda}$, accurate to within 0.001. (See the Special Note, above.)

3. Suppose it is believed that $Y = X^\beta + \text{error}$, with β unknown. Suppose we observe $n = 5$ pairs of observations: $(3, 5)$, $(4, 10)$, $(8, 21)$, $(10, 35)$, and $(11, 40)$. Write a program in R that uses the Illinois Method to find the least-squares estimate of β, accurate to within 0.001. (See the Special Note, above.)

4. Suppose it is believed that $Y = \delta\, X^\beta + \text{error}$, with δ and β both unknown. Suppose we observe the same $n = 5$ pairs of observations as in the previous question: $(3, 5)$, $(4, 10)$, $(8, 21)$, $(10, 35)$, and $(11, 40)$. Write a program in R that uses the Gradient Descent Method, with retraction, to find the least-squares estimate of δ and β. (See the Special Note, above. And, remember that minimising a function requires $\alpha < 0$.)