STA3431H (Monte Carlo Methods), Winter 2009

Homework #1

Due: In class by 2:10 p.m. **sharp** on Monday February 2.

NOTES:

- **Late homeworks, even by one minute, will be penalised!**
- **Include at the top of the first page:** Your name and student number.
- Homework assignments are to be solved by each student individually. You may discuss assignments in general terms with other students, but you must solve it on your own, including doing all of your own computing and writing.
- When writing computer programs for homework assignments:
  - R is the "default" computer programming language, but it is also acceptable to write homework programs in C, C++, Java, Fortran, Pascal, Turing, Cobol, Basic, Mathematica, S-Plus, or SAS, provided you explain that you are doing this. Other languages may be used with prior permission only; e-mail the instructor to enquire.
  - You should hand in both the complete source code and the program output.
  - Programs must be clearly explained with comments etc. so they are easy to follow.

The assignment:

1. Consider the *Buffon needle* experiment as described in class.

(a) Prove that the probability the needle touches a line is $2\ell/w\pi$. [Note: this is essentially done in the lecture notes, you just need to understand them and write the proof up in your own way.]

(b) Suppose instead that $\ell > w$, and that one end of the needle is fixed to a point on a line while the other end is spun in a random direction. Now what is the probability that the needle touches a line (not counting the line under the fixed end)?

2. Write a computer program to compute a Monte Carlo estimate (including standard error) of $\mathbf{E}(|Z|^Y)$, where $Z \sim \text{Normal}(0,1)$ and $Y \sim \text{Exponential}(3)$ are independent, without using any built-in functions for random number generation (e.g. runif, rnorm, etc.). That is, you should just use basic computer commands like variable assignment, arithmetic, arrays, for, if, subroutines/functions, etc., together with your own uniform pseudorandom number generator (of your choice) and your own normal-distribution transformation and exponential-distribution transformation, and your own Monte Carlo routine.

3. **(a)** Describe two different Monte Carlo algorithms for computing

$$I \equiv \int_4^\infty \int_{-\infty}^\infty x^{-|y|^3-2}\, dy\, dx\,.$$

(b) Run each of them to compute an estimate of I together with its standard error. (This time, you may use built-in random number generation for standard distributions.)

(c) Discuss which of the two Monte Carlo algorithms is "better" and why.