STA3431H (Monte Carlo Methods), Winter 2010

Homework #1

Due: In class by 2:10 p.m. **sharp** on Monday February 8.

NOTES:

- **Late homeworks, even by one minute, will be penalised!**
- **Include at the top of the first page:** Your name and student number.
- Homework assignments are to be solved by each student individually. You may discuss assignments in general terms with other students, but you must solve it on your own, including doing all of your own computing and writing.
- When writing computer programs for homework assignments:
  - R is the "default" computer programming language, but it is also acceptable to use other languages with prior permission.
  - You should hand in both the complete source code and the program output.
  - Programs must be clearly explained with comments etc. so they are easy to follow.

The assignment:

1. Write a computer program to compute a Monte Carlo estimate (including standard error) of $\mathbf{E}(|Z|/(1+Y))$, where $Z \sim \text{Normal}(0,1)$ and $Y \sim \text{Exponential}(3)$ are independent, without using any built-in functions for random number generation (e.g. runif, rnorm, etc.). That is, you should just use basic computer commands like variable assignment, arithmetic, log/exp/sin/cos, for, if, etc., together with your own uniform pseudorandom number generator (of your choice) and your own normal-distribution transformation and exponential-distribution transformation, and your own Monte Carlo routine, and your own computation of mean and of standard error.

2. Describe and run any two different Monte Carlo algorithms for computing

$$I \equiv \int_4^\infty \int_{-\infty}^\infty x^{-|y|^2-3}\, dy\, dx\,.$$

Discuss which of the two Monte Carlo algorithms is "better" and why.

Now, let a, b, c, and d be the last four digits of your student number. (So, for example, if your student number were 840245070*, then $a = 5$, $b = 0$, $c = 7$, and $d = 0$.)

3. Identify the values of a, b, c, and d.

Now, let $g : \mathbf{R}^5 \to [0, \infty)$ be the function defined by

$$g(x_1, x_2, x_3, x_4, x_5) \;=\; (x_1 + x_2)^{a+1}(1 + \cos((b+3)x_3))(e^{(12-c)x_4})|x_4 - x_5|^d \prod_{i=1}^{5} \mathbf{1}_{0<x_i<2}\,,$$

and let $\pi(x_1, x_2, x_3, x_4, x_5) \propto g(x_1, x_2, x_3, x_4, x_5)$ be a five-dimensional density function.

4. Attempt to compute $\mathbf{E}_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using an importance sampler of your choice. Or, if this becomes too difficult to succeed, then explain <u>why</u> it is too difficult.

5. Attempt to compute $\mathbf{E}_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using a rejection sampler of your choice. Or, if this becomes too difficult to succeed, then explain <u>why</u> it is too difficult.

6. Attempt to compute $\mathbf{E}_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using an MCMC algorithm of your choice, including some discussion of accuracy, standard errors, etc.

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* (Historical note: this was the instructor's actual student number when he was a UofT undergrad in 1984–88.)