STA3431H (Monte Carlo Methods), Winter 2010

Homework #1

Due: In class by 2:10 p.m. **sharp** on Monday February 8.

NOTES:

- Late homeworks, even by one minute, will be penalised!
- **Include at the top of the first page:** Your name and student number.
- Homework assignments are to be solved by each student **individually.** You may discuss assignments in general terms with other students, but you must solve it on your own, including doing all of your own computing and writing.
- When writing computer programs for homework assignments:
  - R is the ”default” computer programming language, but it is also acceptable to use other languages with prior permission.
  - You should hand in both the complete source code and the program output.
  - Programs must be clearly explained with comments etc. so they are easy to follow.

The assignment:

1. Write a computer program to compute a Monte Carlo estimate (including standard error) of $E(|Z|/(1 + Y))$, where $Z \sim \text{Normal}(0, 1)$ and $Y \sim \text{Exponential}(3)$ are independent, **without** using any built-in functions for random number generation (e.g. runif, rnorm, etc.). That is, you should just use basic computer commands like variable assignment, arithmetic, log/exp/sin/cos, for, if, etc., together with your own uniform pseudorandom number generator (of your choice) and your own normal-distribution transformation and exponential-distribution transformation, and your own Monte Carlo routine, and your own computation of mean and of standard error.

2. Describe and run any **two different** Monte Carlo algorithms for computing

$$I \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^{-|y|^{2-3}} \, dy \, dx.$$

Discuss which of the two Monte Carlo algorithms is “better” and why.
Now, let $a$, $b$, $c$, and $d$ be the last four digits of your student number. (So, for example, if your student number were 840245070*, then $a = 5$, $b = 0$, $c = 7$, and $d = 0$.)

3. Identify the values of $a$, $b$, $c$, and $d$.

Now, let $g : \mathbb{R}^5 \to [0, \infty)$ be the function defined by

$$g(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2)^{a+1}(1 + \cos((b + 3)x_3))(e^{(12 - c)x_4})|x_4 - x_5|^d \prod_{i=1}^{5} 1_{0 < x_i < 2},$$

and let $\pi(x_1, x_2, x_3, x_4, x_5) \propto g(x_1, x_2, x_3, x_4, x_5)$ be a five-dimensional density function.

4. Attempt to compute $E_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using an importance sampler of your choice. Or, if this becomes too difficult to succeed, then explain why it is too difficult.

5. Attempt to compute $E_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using a rejection sampler of your choice. Or, if this becomes too difficult to succeed, then explain why it is too difficult.

6. Attempt to compute $E_\pi[(X_1 - X_2)/(1 + X_3 + X_4 X_5)]$ by using an MCMC algorithm of your choice, including some discussion of accuracy, standard errors, etc.

* (Historical note: this was the instructor’s actual student number when he was a UofT undergrad in 1984–88.)