STA447/2006 Midterm #1, February 7, 2019

(135 minutes; 4 questions; 7 pages; total points = 50)

FAMILY NAME: ______________________ **GIVEN NAME(S):** ______________________

STUDENT #: ______________________ **SIGNATURE:** ______________________

Class (circle one): STA447 STA2006

Do not open this booklet until told to do so. Answer all questions.
Aids allowed: NONE. You may use results from class, with explanation.
Point values for each question are indicated in [square brackets].
You should explain all of your solutions clearly.
You may continue on the back of the page if necessary (write "OVER").
Scrap paper is included at the end of this test.

DO NOT WRITE BELOW THIS LINE.

Question	Score
1(a)	/2
1(b)	/5
1(c)	/3
1(d)	/6
1(e)	/3
2(a)	/4
2(b)	/3
2(c)	/3

Question	Score
2(d)	/3
2(e)	/3
3(a)	/3
3(b)	/3
3(c)	/3
4	/6
TOTAL:	/50

1. Consider a Markov chain with state space $S = \{1, 2, 3\}$, and transition probabilities $p_{12} = 1/2$, $p_{13} = 1/2$, $p_{21} = 1/3$, $p_{23} = 2/3$, and $p_{31} = 1$, otherwise $p_{ij} = 0$.

(a) [**2**] Compute $p_{11}^{(2)}$.

(b) [**5**] Find a probability distribution π which is stationary for this chain.

(c) [**3**] Determine if the chain is reversible with respect to π.

1. (continued)

(d) [**6**] Determine (with explanation) which of the following statements are true and which are false: (i) $\lim_{n\to\infty} p_{13}^{(n)} = \pi_3$. (ii) $\lim_{n\to\infty} \frac{1}{2}[p_{13}^{(n)} + p_{13}^{(n+1)}] = \pi_3$. (iii) $\lim_{n\to\infty} \frac{1}{n} \sum_{\ell=1}^{n} p_{13}^{(\ell)} = \pi_3$.

(e) [**3**] Determine (with explanation) whether or not $\sum_{n=1}^{\infty} p_{13}^{(n)} = \infty$.

2. Consider a Markov chain with state space $S = \{1, 2, 3, 4\}$ and transition matrix:

$$P = \begin{pmatrix} 1/4 & 1/2 & 1/8 & 1/8 \\ 0 & 1/3 & 0 & 2/3 \\ 0 & 0 & 1 & 0 \\ 0 & 4/5 & 0 & 1/5 \end{pmatrix}$$

(a) [**4**] Specify (with explanation) which states are recurrent, and which are transient.

(b) [**3**] Compute f_{24}.

2. (continued)

(c) [**3**] Compute f_{14}.

(d) [**3**] Determine whether or not $\sum_{n=1}^{\infty} p_{24}^{(n)} = \infty$.

(e) [**3**] Determine whether or not $\sum_{n=1}^{\infty} p_{14}^{(n)} = \infty$.

3. For each of the following sets of conditions, either provide (with explanation) an example of a state space S and Markov chain transition probabilities $\{p_{ij}\}_{i,j \in S}$ such that the conditions are satisfied, or prove that no such a Markov chain exists.

(a) [**3**] There is $k \in S$ having period 1, and $\ell \in S$ having period 3.

(b) [**3**] The chain is irreducible, and there are distinct states $i, j, k, \ell \in S$ such that $f_{ij} = 1$, and $\sum_{n=1}^{\infty} p_{k\ell}^{(n)} < \infty$.

(c) [**3**] There are distinct states $i, j, k \in S$ with $f_{ij} = 1/3$, $f_{jk} = 1/4$, and $f_{ik} = 1/20$.

4. **[6]** Prove the Equal Periods Lemma, i.e. prove that if $i \leftrightarrow j$, and t_i is the period of state i, and t_j is the period of state j, then $t_i = t_j$. [Note: You cannot <u>use</u> the Equal Periods Lemma or any later results from class to prove this, you have to prove it yourself.]

[END OF EXAMINATION; total points = 50]

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