STA447/2006 Midterm #1, February 6, 2020

(135 minutes; 6 questions; 7 pages; total points = 50)

FAMILY NAME: ____________________ **GIVEN NAME(S):** ____________________

STUDENT #: ____________________ **SIGNATURE:** ____________________

Class (circle one): STA447 STA2006

- **Do not open this booklet until told to do so. Answer all questions.**
- **Aids allowed: NONE. You may use results from class, with explanation.**
- You should **explain** all of your solutions clearly.
- Point values for each question are indicated in [square brackets].
- You may continue on the back of the page if necessary (write "OVER").
- **Scrap paper is included at the end of this test (and may be detached).**

DO NOT WRITE BELOW THIS LINE.

Question	Score
1(a)	/2
1(b)	/2
1(c)	/3
2(a)	/3
2(b)	/3
2(c)	/4
3(a)	/3
3(b)	/3
3(c)	/3

Question	Score
4(a)	/3
4(b)	/5
5(a)	/3
5(b)	/3
5(c)	/3
6	/7
TOTAL:	/50

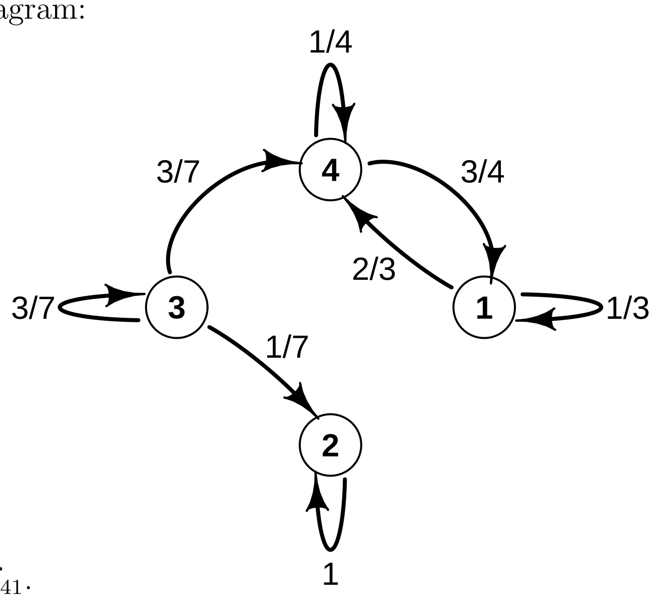
1. Consider a Markov chain with state space $S = \{1, 2, 3\}$, and transition probabilities $p_{12} = 1/2$, $p_{13} = 1/2$, $p_{21} = 1/4$, $p_{23} = 3/4$, and $p_{31} = 1$, otherwise $p_{ij} = 0$.

(a) [**2**] Draw a <u>diagram</u> of this Markov chain.

(b) [**2**] Compute $p_{11}^{(2)}$.

(c) [**3**] Determine (with explanation) whether or not $\sum_{n=1}^{\infty} p_{12}^{(n)} = \infty$.

2. Consider a Markov chain with state space $S = \{1, 2, 3, 4\}$ and transition probabilities as in the following diagram:



(a) **[3]** Compute f_{41}.

(b) **[3]** Compute f_{31}.

(c) **[4]** Compute $\sum_{n=1}^{\infty} p_{33}^{(n)}$, and determine if state 3 is recurrent or transient.

3. For each of the following sets of conditions, either provide (with explanation) an example of a state space S (which contains states 1 and 2, but might also contain other states too), and Markov chain transition probabilities $\{p_{ij}\}_{i,j \in S}$, such that the conditions are satisfied, or prove that no such a Markov chain exists.

(a) [**3**] The chain is <u>irreducible</u>, and $\sum_{n=1}^{\infty} p_{12}^{(n)} < \infty$, and $f_{12} = 1$.

(b) [**3**] $\sum_{n=1}^{\infty} p_{11}^{(n)} = \infty$, and $p_{21} > 0$, but $f_{21} < 1$.

(c) [**3**] For all $n \in \mathbf{N}$, $p_{12}^{(n)} \geq 1/3$ and $p_{21}^{(n)} \geq 1/5$, and state 2 is <u>transient</u>.

4. Suppose a Markov chain has distinct states $i, j \in S$, with i recurrent, and j transient. (Of course, S might also contain other states too.)

(a) [**3**] Show by example that it is possible that $j \to i$.

(b) [**5**] Prove that it is impossible that $i \to j$.

5. Consider a Markov chain having states $i, j \in S$, such that $\mathbf{P}_i[N(j) = \infty] = 1/3$. (Of course, S might also contain other states too.)

(a) [**3**] Prove that any such chain <u>cannot</u> have $i = j$.

(b) [**3**] Prove that any such chain <u>cannot</u> be irreducible.

(c) [**3**] Provide (with explanation) a valid <u>example</u> of such a chain.

6. [7] Consider a Markov chain with state space $S = \{1, 2, 3\}$, and transition probabilities

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/3 & 2/3 \\ 1/4 & 0 & 3/4 \end{pmatrix}$$

Either compute $\lim_{n\to\infty} p_{12}^{(n)}$, or prove that the limit does not exist.

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