“Struck by Lightning” Supplementary Materials

Mathematical exercise about the game “craps”

Recall that the game “craps” is played as follows. A player rolls two fair six-sided dice. If their sum is 7 or 11, the player wins right away. If their sum is 2, 3, or 12, the player loses right away. If their sum is any other number, this number becomes the “point”. The player then repeatedly rolls two dice until their sum equals the point (in which case the player wins) or equals 7 (in which case the player loses). Once a game has been won or lost, the next game starts over from the beginning.

You will be assigned to new groups. Introduce yourselves to each other before you begin. Then, working cooperatively with your group, answer the following questions. (Note: do not refer to the book or the internet while working on this assignment!)

1. Make sure everyone in your group understands the rules of craps.

2. As a group, take two dice. Have each member of the group play the game of craps at least 10 times (starting over each time), keeping track of how many times each member wins and loses.

3. As a group, make your best guess of the probability of winning at craps. Write your guess, together with all the group member names, on an index card and hand it in to the instructor. (There will later be a PRIZE for the group that comes closest to the true probability.)

Next, compute the probability of winning at craps, as follows:

4. Compute the probability p_i of rolling the sum i on the *first* roll of the two dice, for $i = 2, 3, 4, \ldots, 12$. (Hint: you might want to first make a 6×6 chart of all possible pairs of die values.)

5. Compute the probability q_i that you win at craps *if* we get the result i on the first roll of the two dice, for $i = 2, 3, 4, \ldots, 12$. [Hint: For example, $q_2 = 0$ because if you roll a 2 the first time then you lose. Similarly $q_7 = 1$. Thus, q_2, q_3, q_7, q_{11}, and q_{12} are all easy. The other values of q_i are harder. For example, q_4 is the probability that, if you repeatedly roll two dice, you will get 4 before you get 7, ignoring any rolls that are neither 4 nor 7. How can you compute this probability?]

6. Now that you know all the values of p_i and q_i, compute the overall probability, p, of winning at craps. [Hint: This is just a question of combining all the p_i and q_i together in the right way. Remember that if you do win at craps, you had to get *some* result on the first roll, and then win after that. So, what is the final formula?]

If you have time, consider the following further questions:

7. How does this probability p (of winning at craps) compare to 50%? Why is this an important question? [Hint: pretend you're at a casino, and play craps repeatedly.]

8. Suppose that in one out of every n games, you could *cheat* at craps so that you will win for sure (instead of just having a certain probability of winning). For what values of n would this allow you to make a long-run profit by repeatedly playing craps? Explain.

9. Suppose the first die is *weighted*, so that the probability it shows 6 is 1/2, while the probability it shows any other number (from 1 to 5) is 1/10. (The second die remains fair, i.e. with probability 1/6 of showing each number.) What is the probability of winning at craps with these new dice? [Hint: follow the same steps as above, but the values of p_i and q_i will now be different.]

10. Suppose instead the dice are weighted as follows. For the first die, the probability it shows 5 is 1/2, while the probability it shows any other number is 1/10. For the second die, the probability it shows 6 is 1/2, while the probability it shows any other number is 1/10. What is the probability of winning at craps with *these* dice?

11. If you have time, make up a few *other* games, which still involve repeatedly rolling two fair dice, but which have (at least slightly) different rules from craps. For each game, compute its probability of winning. Try to find a game which has probability *just over* 70% of winning.