<u>**"Struck by Lightning" Supplementary Materials**</u>

<u>Group Exercise about Poker Probabilities</u>

Recall that an *ordinary 52-card deck* of cards consists of 52 cards, of which 13 each are Clubs, Diamonds, Hearts, and Spades. Recall that an *ordinary poker hand* consists of 5 cards, chosen uniformly at random from an ordinary 52-card deck. Recall that a poker hand is a *flush* if all 5 cards are of the same suit, i.e. either all Clubs, or all Diamonds, or all Hearts, or all Spades.

1. Compute the probability that a given ordinary poker hand is a flush.

2. In some poker games, the cards are dealt out a few at a time, rather than all at once. Suppose a player already has three cards, and they are all Clubs. The player will then be dealt two more cards, chosen uniformly at random from the remainder of the deck. Compute the probability that they will end up with a flush. How does this probability compare with your answer to Question 1?

3. In some poker games (e.g. "Five Card Stud"), a player gets to see some of their opponents cards (the "up cards"), before the hand is complete. Suppose a game is such that each player has been dealt three cards, of which two are "up cards". Each player will later be dealt two more cards, chosen uniformly at random from the remainder of the deck. Suppose there are five players in the game. Suppose that one player ("Player #1") has all three cards Clubs.

(a) Suppose further that, of all the up cards of all the other four players, none of them are Clubs. Compute the probability that Player #1 will end up with a flush.

(b) Suppose instead that, of all the up cards of all the other four players, *all* of them are Clubs. Compute the probability that Player #1 will end up with a flush.

(c) Suppose the situation (because of the betting so far) is such that Player #1 will fold (i.e., drop out of the game) unless they have at least a 3% chance of getting a flush. Compute the smallest number of Clubs among the up cards of all the other four players, such that Player #1 will fold.

(d) What conclusions can be drawn from this question, regarding an actual game of poker?

4. In some poker games (e.g. "Seven Card Stud"), players get *more* than five cards, and they then get to *choose* which five cards count as their final poker hand. In this case, a hand is a flush if *at least 5* of its cards are of the same suit. Suppose in a given game, each player is dealt a total of 7 cards. Compute the probability that a given player will obtain a flush.

To continue, recall that an ordinary poker hand is a *straight* if it consists of 5 cards whose face values are in succession. For example: Ace-2-3-4-5, or 3-4-5-6-7, or 8-9-10-Jack-Queen, or 10-Jack-Queen-King-Ace are all straights. (Note that it is *not* permitted to "go around the corner", so that e.g. Queen-King-Ace-2-3 is *not* a straight.)

5. Compute the probability that an ordinary poker hand is a straight.

6. Suppose a player already has three cards, and their face values are 4, 5, and 6, respectively. The player will then be dealt two more cards, chosen uniformly at random from the remainder of an ordinary 52-card deck. Compute the probability that they will end up with a straight.

7. Suppose a player already has three cards, and their face values are 4, 5, and 8, respectively. The player will then be dealt two more cards, again chosen uniformly at random from the remainder of an ordinary 52-card deck. Compute the probability that they will end up with a straight.

8. Compare the probabilities you have computed in all the previous questions. How might these comparisons affect someone playing an actual game of poker?

For the remaining questions, we will generalise the ordinary 52-card deck to an n-card deck (where we will then let $n \to \infty$). We will do this in three different ways, as follows. (Note that in each case, if $n = 52$, then we have an ordinary 52-card deck.)

(I) The deck consists of n cards (where n is a multiple of 4), of which $n/4$ each are Clubs, Diamonds, Hearts, and Spades. [For example, perhaps several ordinary decks have been mixed together.]

(II) The deck consists of n cards (where n is a multiple of 13), of which 13 belong to each of $n/13$ different suits.

(III) The deck consists of n cards (where n is at least 44), of which 13 each are Diamonds, Hearts, and Spades, and the remaining $n - 39$ are Clubs.

9. For each of decks (I), (II), and (III) as above, compute the probability that a given poker hand (consisting as usual of 5 cards chosen uniformly at random from the deck) will be a flush.

10. Compute the limit as $n \to \infty$ of each of the three probabilities in the previous question.

11. Suppose for deck (III) as above, with n a multiple of 52, we consider hands consisting of $5n/52$ cards (instead of 5 cards), and say a hand is a flush if *all* $5n/52$ cards are the same suit. Compute the probability of such a hand being a flush. (You may assume for simplicity that $n > \frac{13 \times 52}{5}$, so that Club flushes are the only possible flushes.) Then compute the limit as $n \to \infty$ of this probability.