

## “Struck by Lightning” Supplementary Materials

### Mathematical exercise about Birthday Matching Probabilities

Working cooperatively with your group, consider the following questions. Assume that each person’s birthday is equally likely to fall on any of 365 different days. (This ignores leap years and uneven birthday distributions, but it is approximately correct.)

First, we consider the distinction between “probability of at least one”, and “expected number” (i.e. “average value”).

1. Suppose each day you count the number of purple-haired people you see, and you obtain the following ten counts: 1, 0, 0, 2, 1, 0, 3, 1, 0, 1. Based on this, what is (a) the probability of seeing at least one purple-haired person each day, and (b) the expected number of purple-haired people you will see each day? [Note: write your probabilities as numbers between 0 and 1, rather than as percentages.]

2. What is the relationship between quantities (a) and (b) in the previous question? Which is larger? By how much? Why?

3. Consider flipping two fair coins. Compute (a) the expected number of heads, and (b) the probability of getting at least one head. Are these two quantities the same? Why or why not? If not, then which one is larger and why?

4. Suppose you and your friend each count purple-haired people each day. Determine if the following statements are true or false: (a) the expected number of purple-haired people that both of you combined see is equal to the sum of the two individual expected numbers; (b) the probability that both of you combined see at least one purple-haired person each day is equal to the sum of the two individual probabilities.

5. The property of the total being equal to the sum of the individual parts is called “linear”. In light of the previous question, is “probability of at least one” linear? Is “expected number” linear?

6. Recall that the probability of winning the Lotto Max jackpot with a single choice of seven numbers is about one chance in 86 million. Suppose one week a total of 30 million independent selections of seven numbers for the Lotto Max are made. (a) What is the expected number of jackpot winners? (b) How is this expected number related to the probability that there will be at least one winner? (Give as much detail as you can.)

Next, we move on to expected numbers of birthday matchings.

7. If there are just two people, what is the probability they have the same birthday?

8. With 23 people, how many pairs of people are there?

9. With 23 people, what is the expected number (i.e., average number) of birthday matchings? [Hint: this is easy if you use the “linear” property as discussed above.]

10. With  $n$  people, how many pairs of people are there?

11. With  $n$  people, what is the expected number of birthday matchings?

12. Check the first three columns of at least three different lines in Table 2.1 on page 16 of the book.

13. If 75 songs are chosen at random from 4,000 songs in total, what is the expected number of matches? How does this relate to the “Musical Mayhem” story on page 17 of the book?

14. If 75 songs are chosen at random from 500 songs in total, what is the expected number of matches?

Next, we consider the probability of getting at least one match.

15. Why is the probability of getting at least one match not the same as the expected number of matches? Which is larger? Explain.

16. If there are just two people, then what is the probability they have *different* birthdays?

17. What is the probability that three people (say, A, B, and C) all have different birthdays? [Hint: For this to happen, first A and B need to have different birthdays. Then C needs to have a birthday different from both A and B.]

18. What is the probability that four people all have different birthdays? [Hint: Generalize the previous solution.]

19. If possible, rewrite the previous answer in terms of “factorials” like  $m! = (m)(m-1)(m-2)\dots(2)(1)$ . [Hint: multiply and divide by the same quantity.]

20. What is the probability that  $n$  people all have different birthdays? [Hint: Find a pattern. You can write your answer as a big product, using “...”.]

21. If possible, rewrite the previous answer in terms of factorials.

22. What is the probability that  $n$  people have at least one birthday match? [Hint: This is easy once you have solved question #20.]

23. Check the fourth columns of the first three different lines in Table 2.1 on page 16 of the book.

Finally, we consider the question of three people all having the same birthday.

24. If there are just three people, then what is the probability that all three have the same birthday?

25. With 23 people, how many different triples of people are there?

26. With 23 people, what is the expected number of triples which all have the same birthday? [Hint: remember that expected values are “linear”.]

27. With  $n$  people, how many different triples of people are there?

28. With  $n$  people, what is the expected number of triples which all have the same birthday?

29. What does the previous answer equal when  $n = 40$ ?  $n = 50$ ?  $n = 60$ ?  $n = 70$ ?  $n = 80$ ?  $n = 90$ ?

30. How large does  $n$  have to be before the expected number of triples which all have the same birthday is more than 0.5?

31. How large do you think  $n$  has to be before there is more than a 50% chance that at least one triple all have the same birthday? (Just give your best guess. Write your guess, together with your group members’ names, on an index card and hand it in to the instructor. There will be prizes for the group that comes the closest.)

32. Do you think it is possible to figure out mathematically the answer to the previous question? Would it be simple? How would you go about it?